

# workshop in particle physics

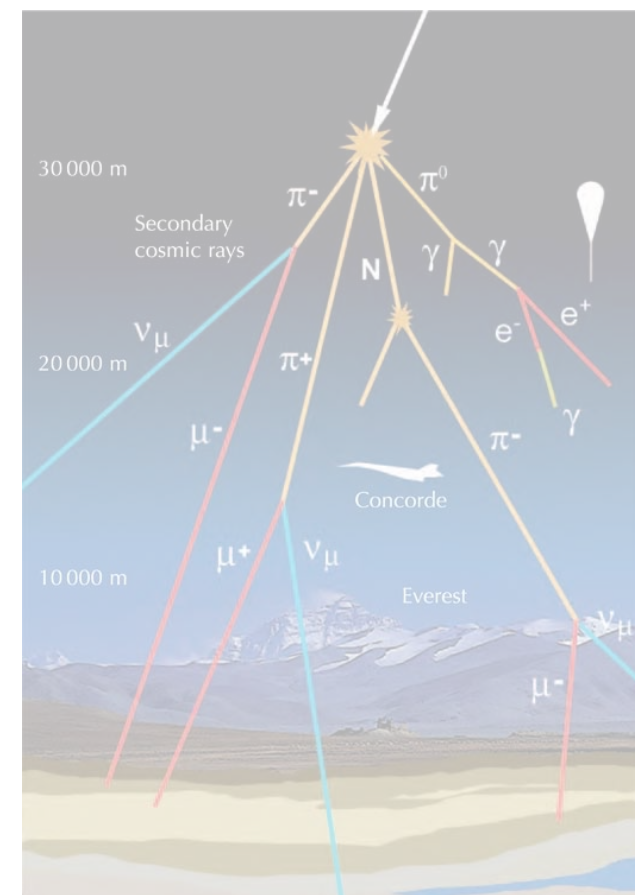
## hadronic showers

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Wisconsin IceCube Particle Astrophysics Center (WIPAC)  
& Department of Astronomy

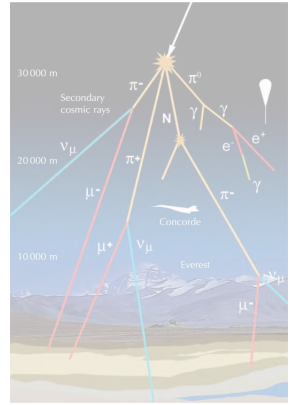
University of Wisconsin - Madison



# lectures outline

## workshop in particle physics

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### 1. fundamental interactions & EM showers

### 2. development of electromagnetic showers

### 3. hadronic showers



# outline

## workshop in particle physics

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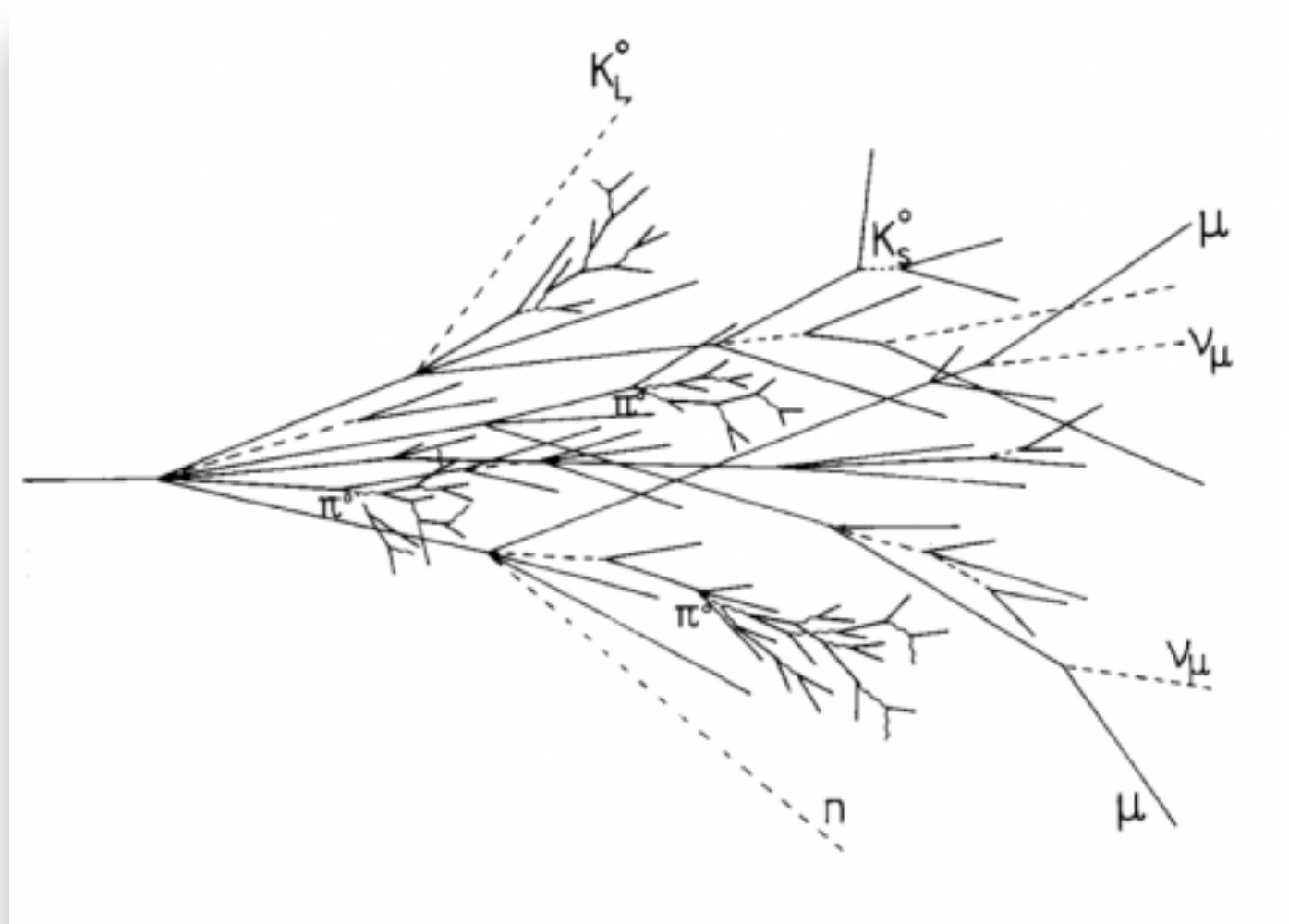
quantum electrodynamics

color confinement: from parton to particles

hadronization

collider experiments feeding cosmic ray physics

inclusive particle flux in the atmosphere



# HADRONIC SHOWERS

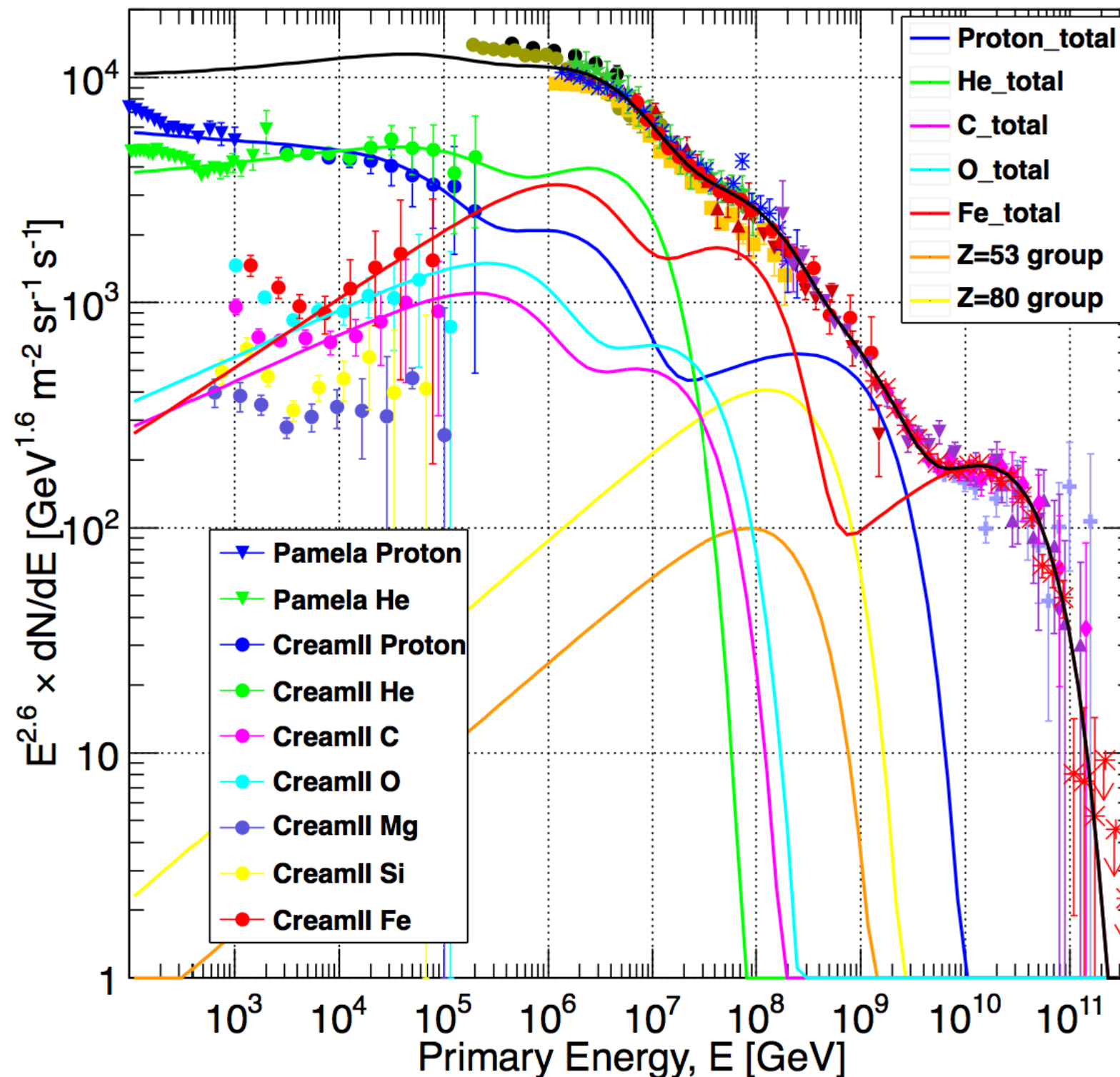
# primary cosmic rays spectrum and composition

shape from **astrophysics**

(source & propagation)

and **particle physics**

(indirect detection)



Gaisser, Stanev, Tilav  
arXiv:1303.3565

# primary cosmic rays

## spectrum and composition

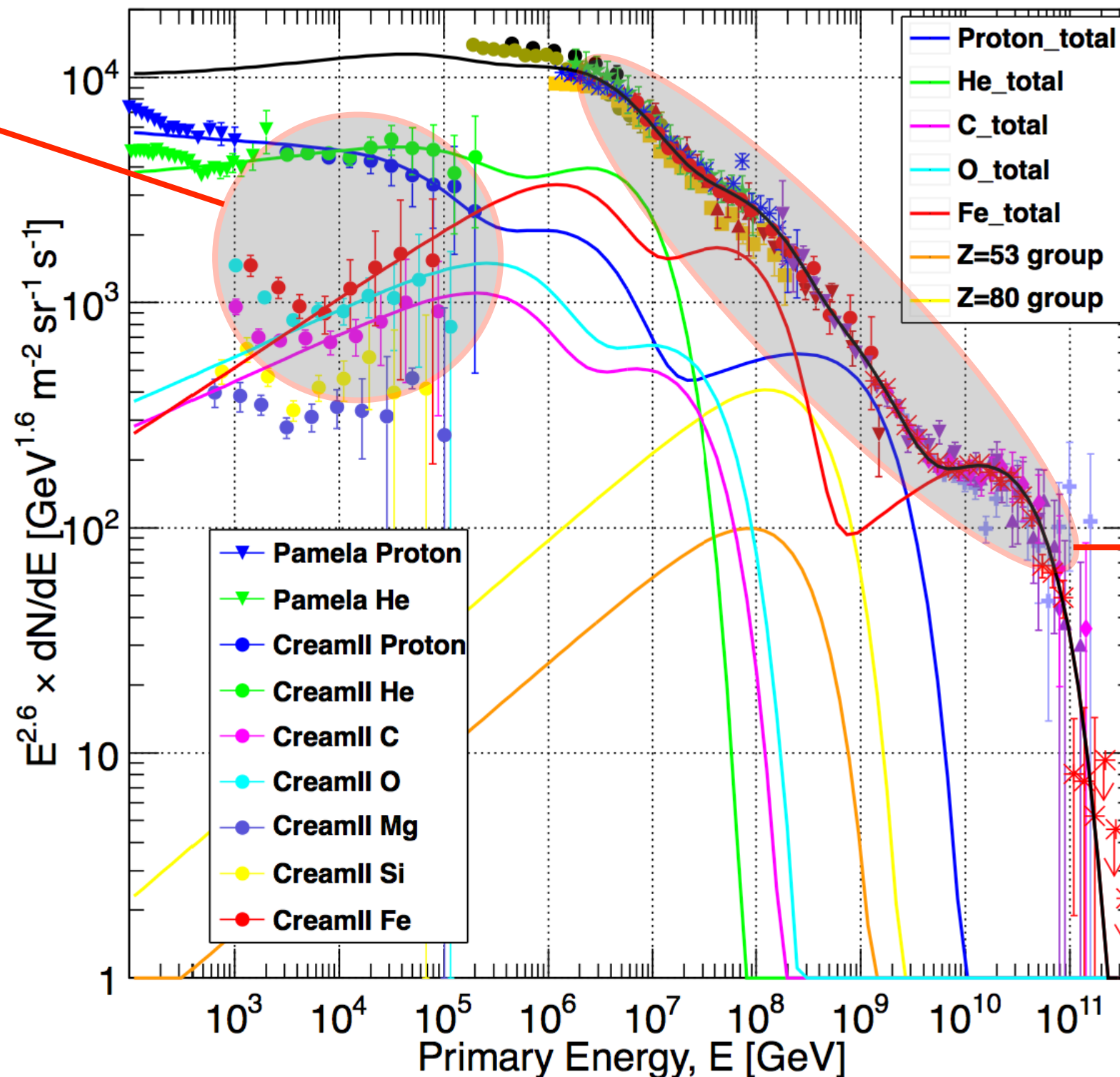
shape from **astrophysics**

(source & propagation)

and **particle physics**

(indirect detection)

**direct  
measurements**



**indirect  
measurements**

Gaisser, Stanev, Tilav  
arXiv:1303.3565

# cosmic rays spectrum

## direct observations



CREAM, ATIC, Bess-Polar  
TRACER, TIGER

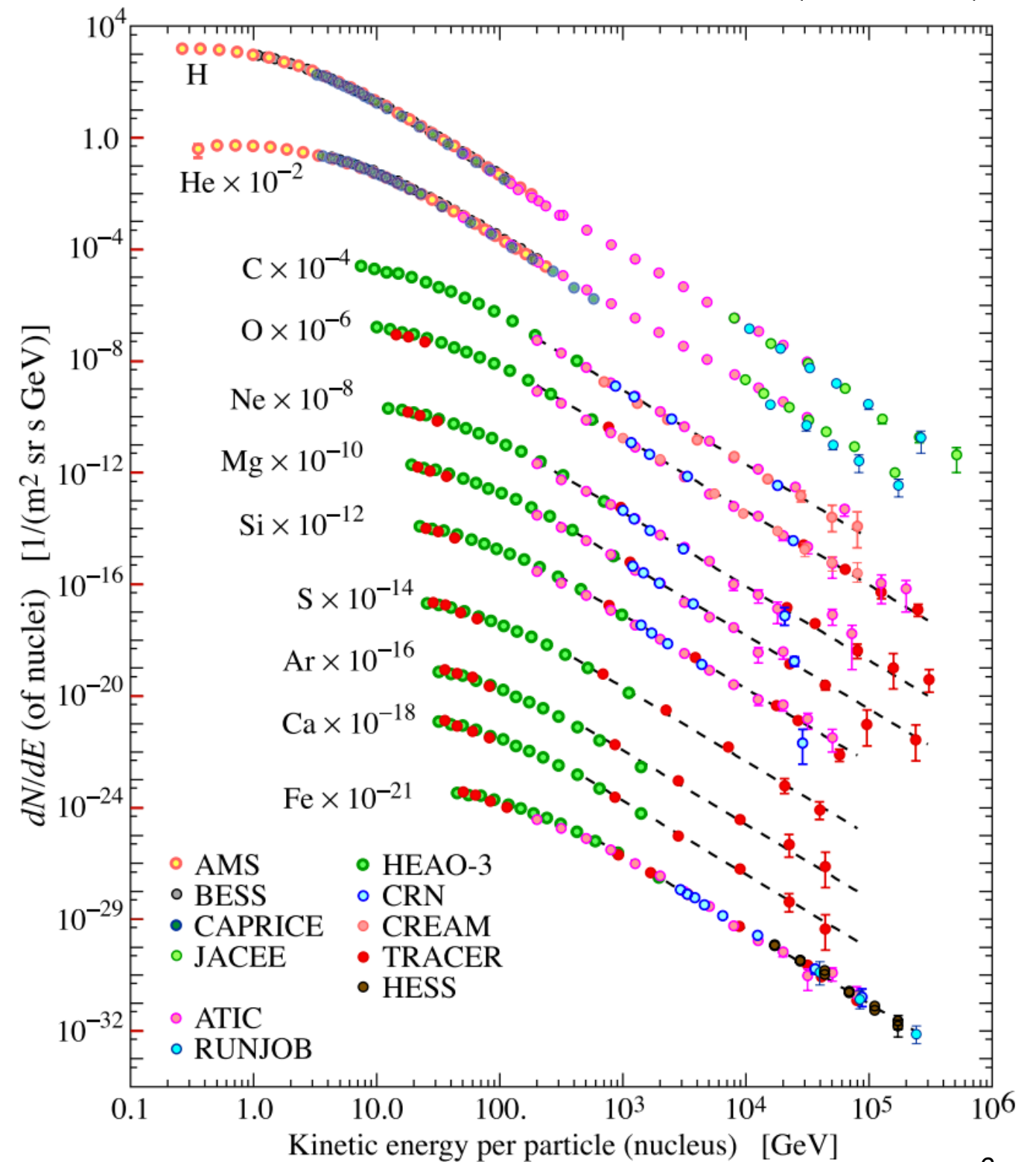


PAMELA, Fermi,  
Gamma-400,...



AMS2, Calet,  
ISS-CREAM, ...

(from PDG)



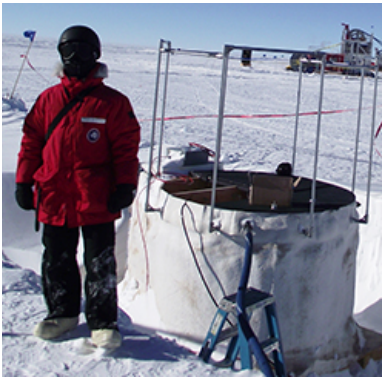


# cosmic rays spectrum

## indirect observations



Tibet Array, ARGO-YBJ

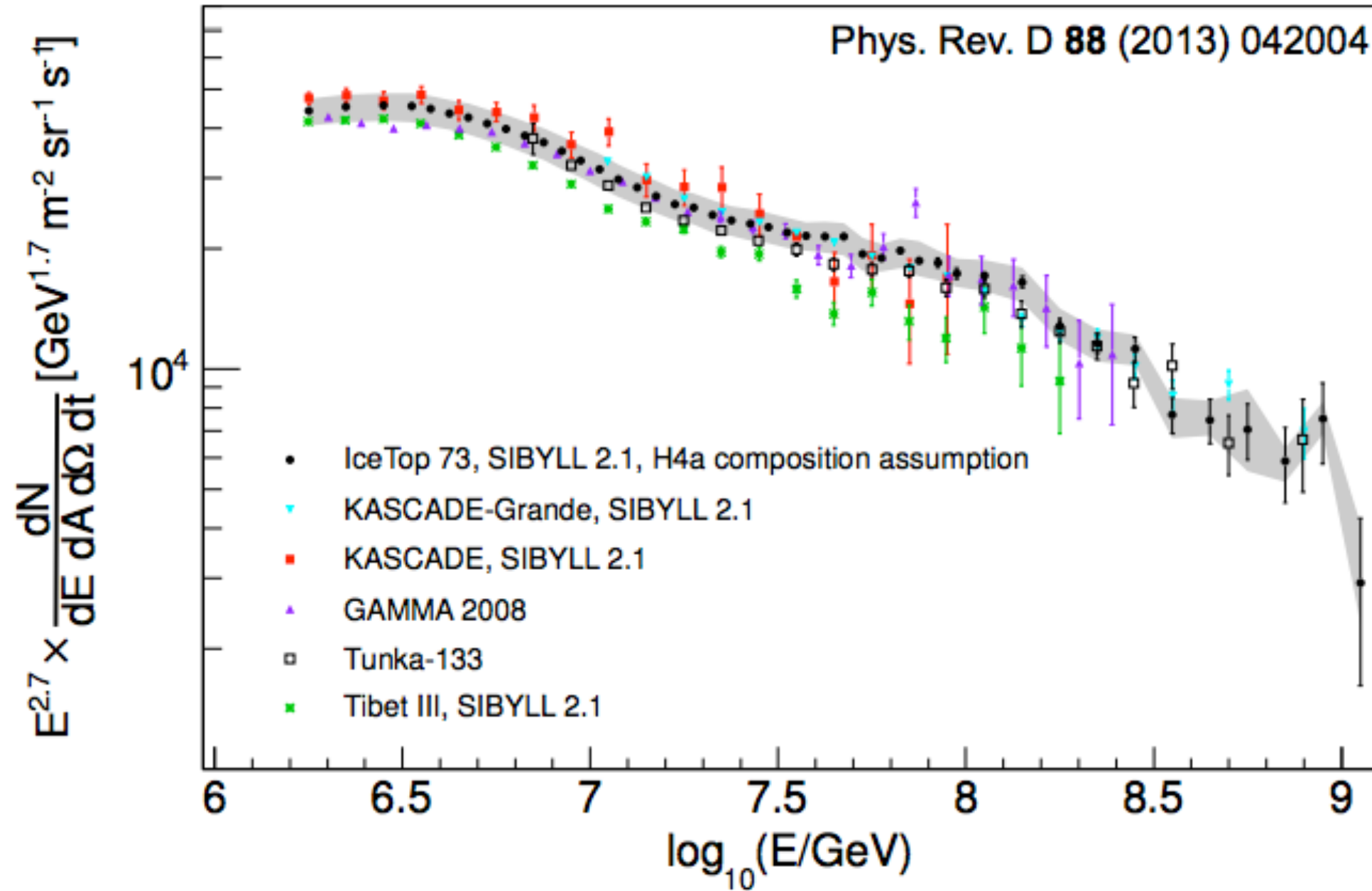


IceTop



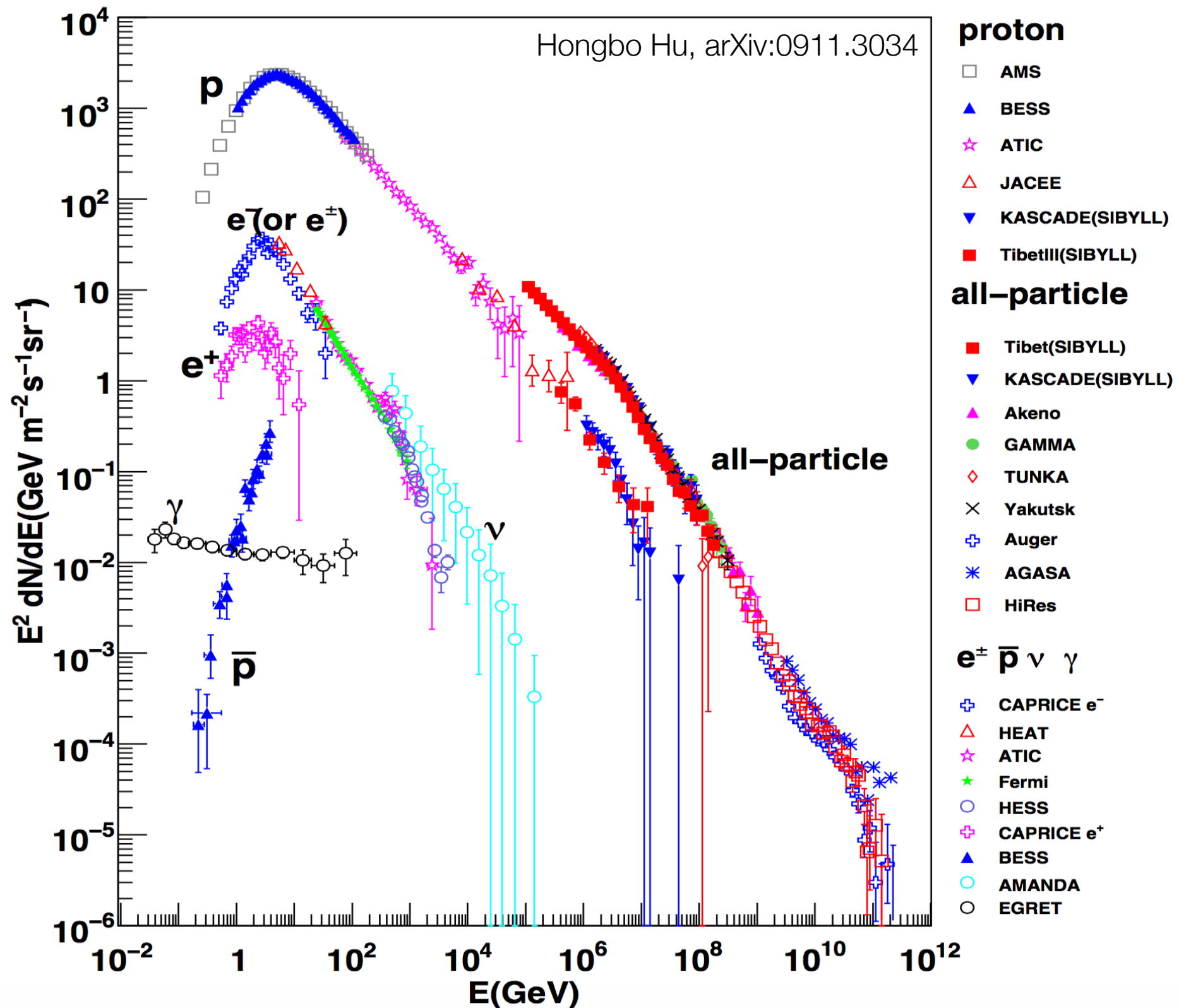
KASCADE-Grande

...



# cosmic rays

bombarding Earth from space

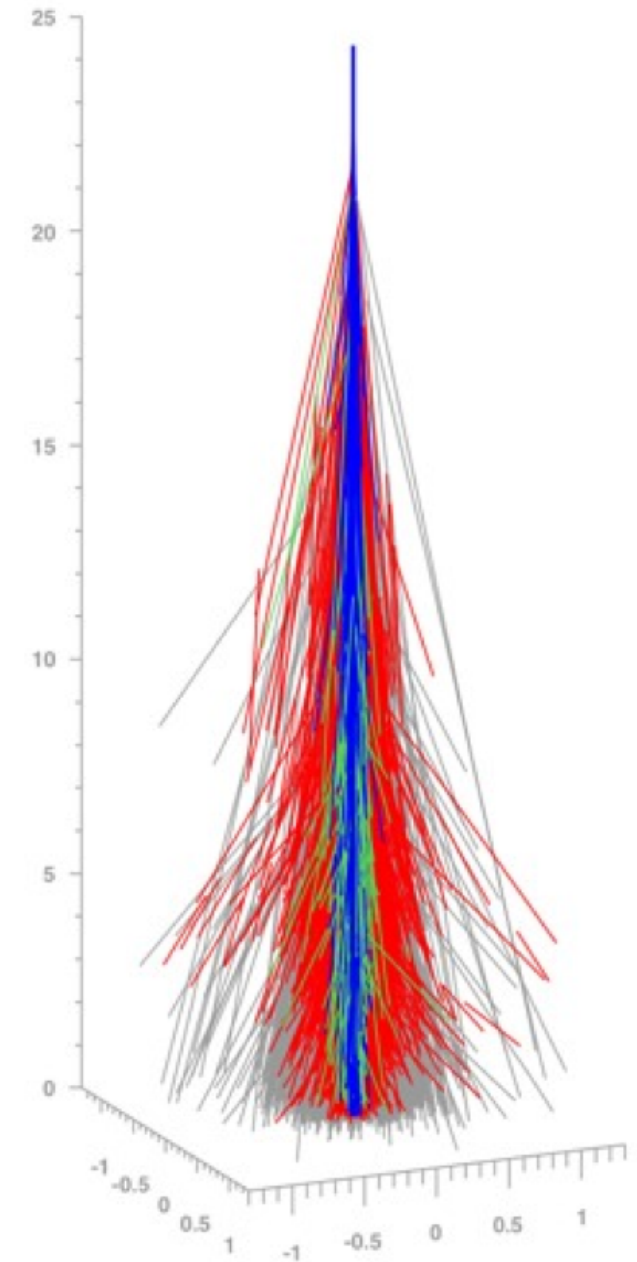
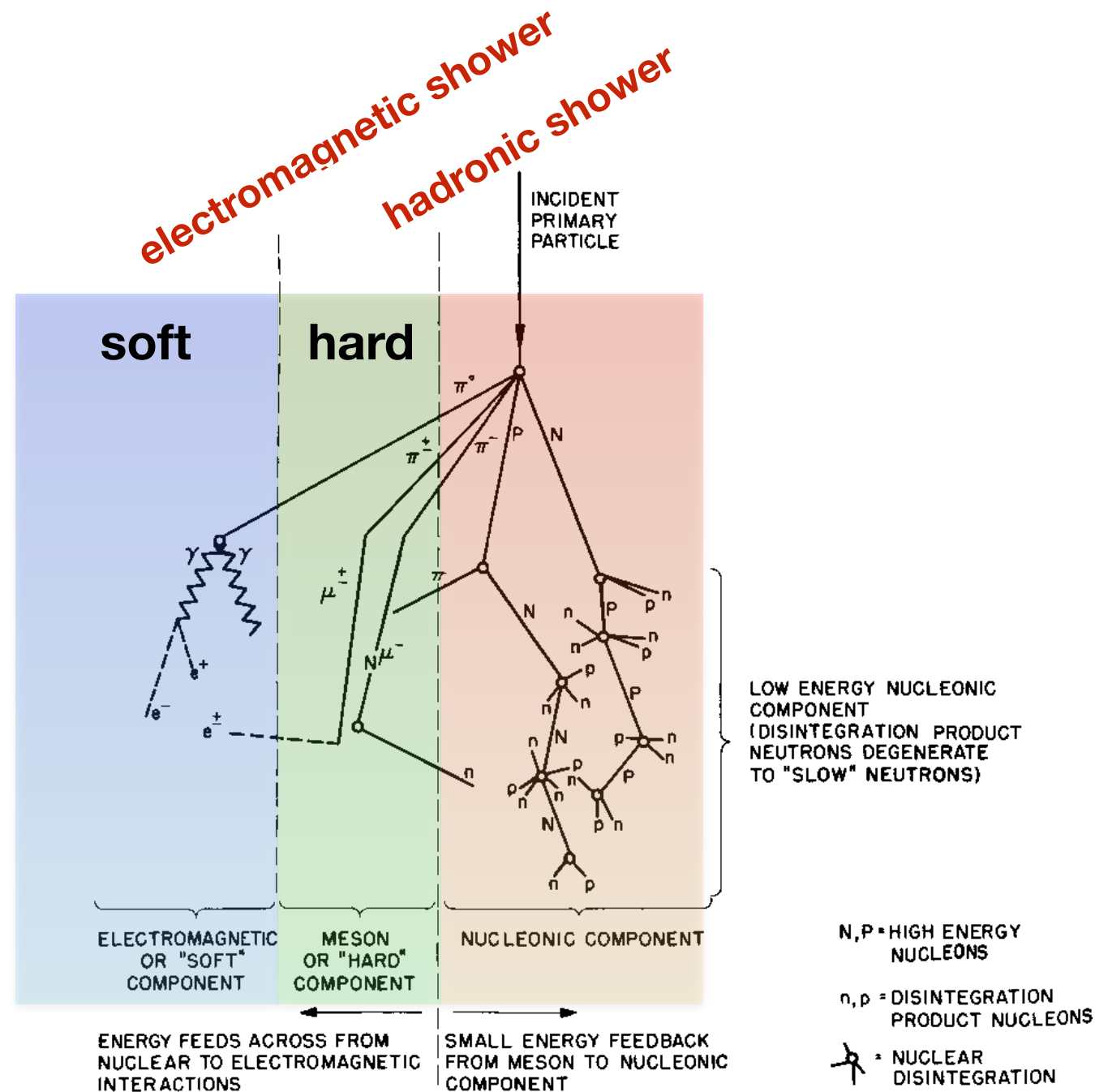


# extensive air showers

## penetrating cosmic radiation

proton-induced  
shower of  $10^{19}$  eV

- atmospheric air showers of particles are extended





# extensive air showers

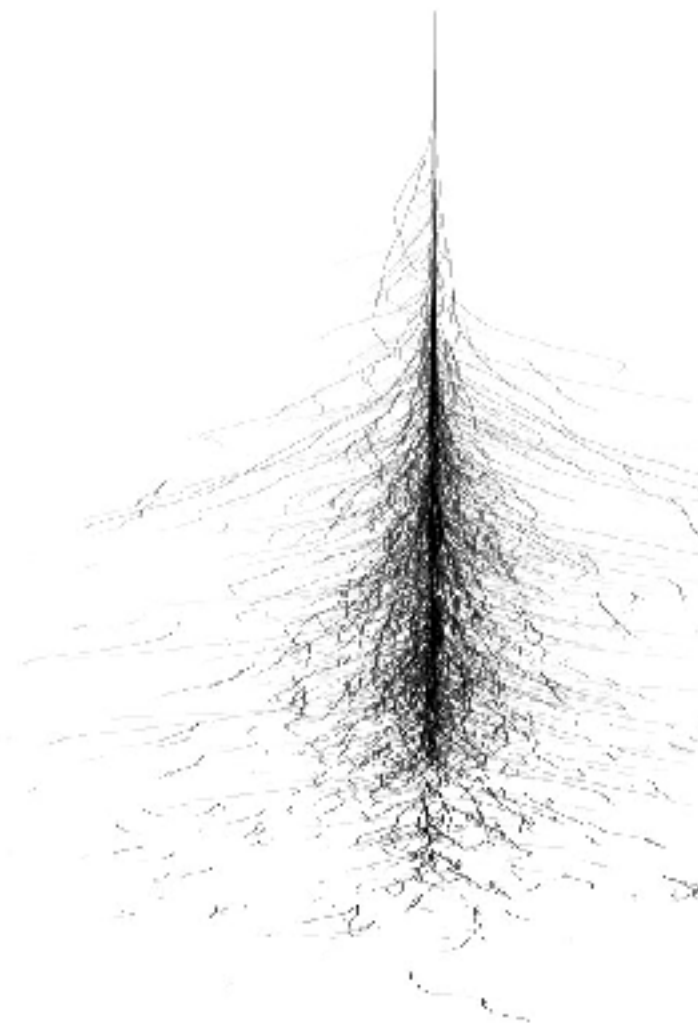
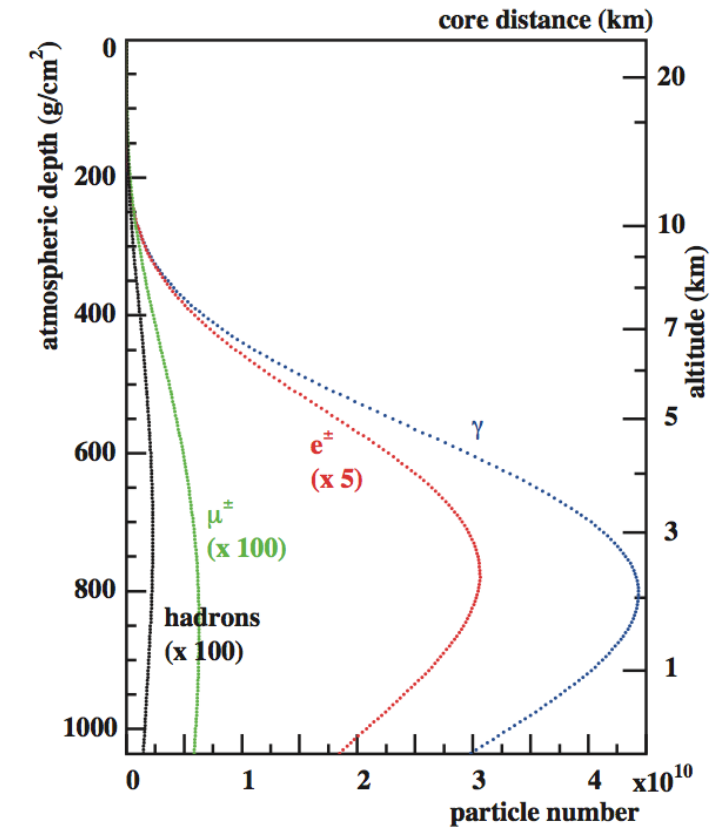
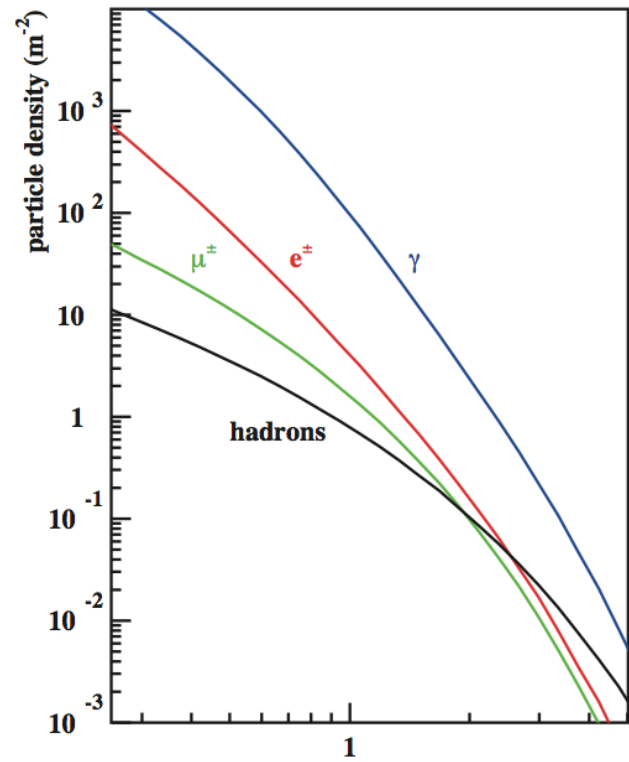
## EM and hadronic showers

topologically complicated

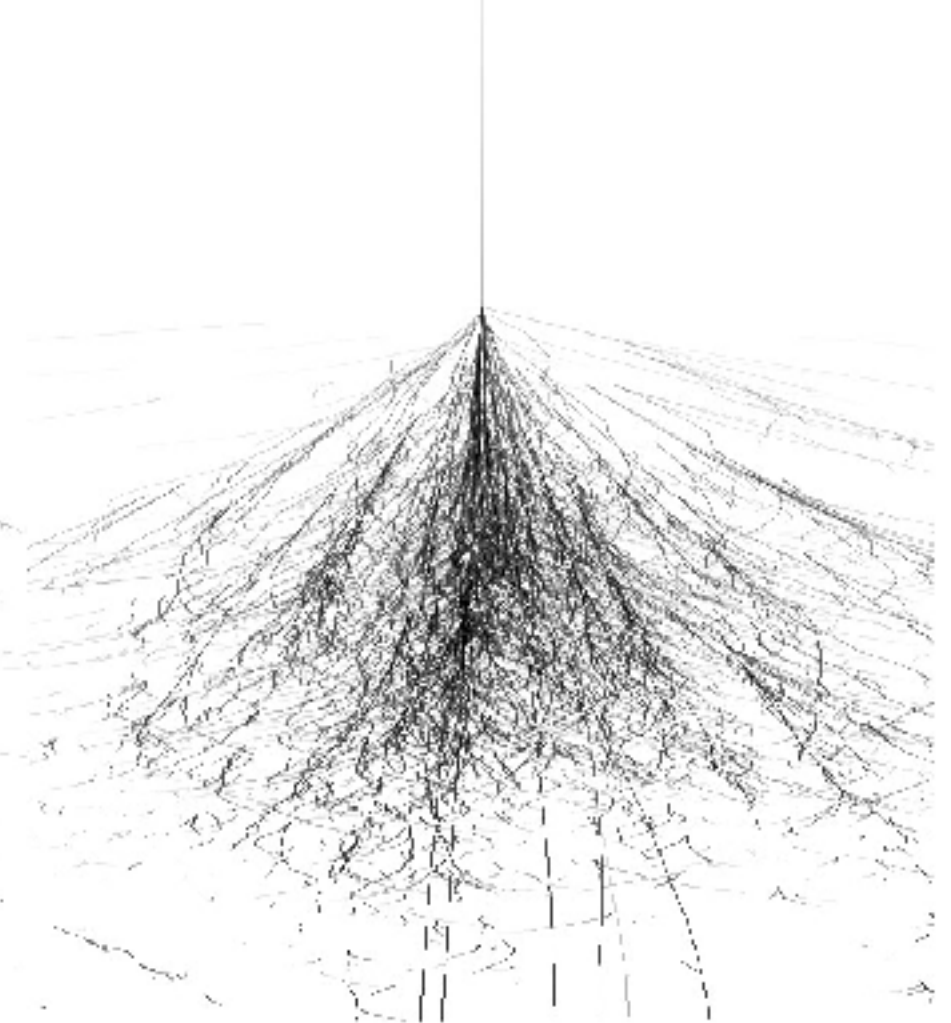
**cannot be described from first principles**

large uncertainties on interaction cross sections  
and hadronization

(RE, Pierog, Heck, ARNPS 2011)



Gamma shower



Hadronic shower

# extensive air showers

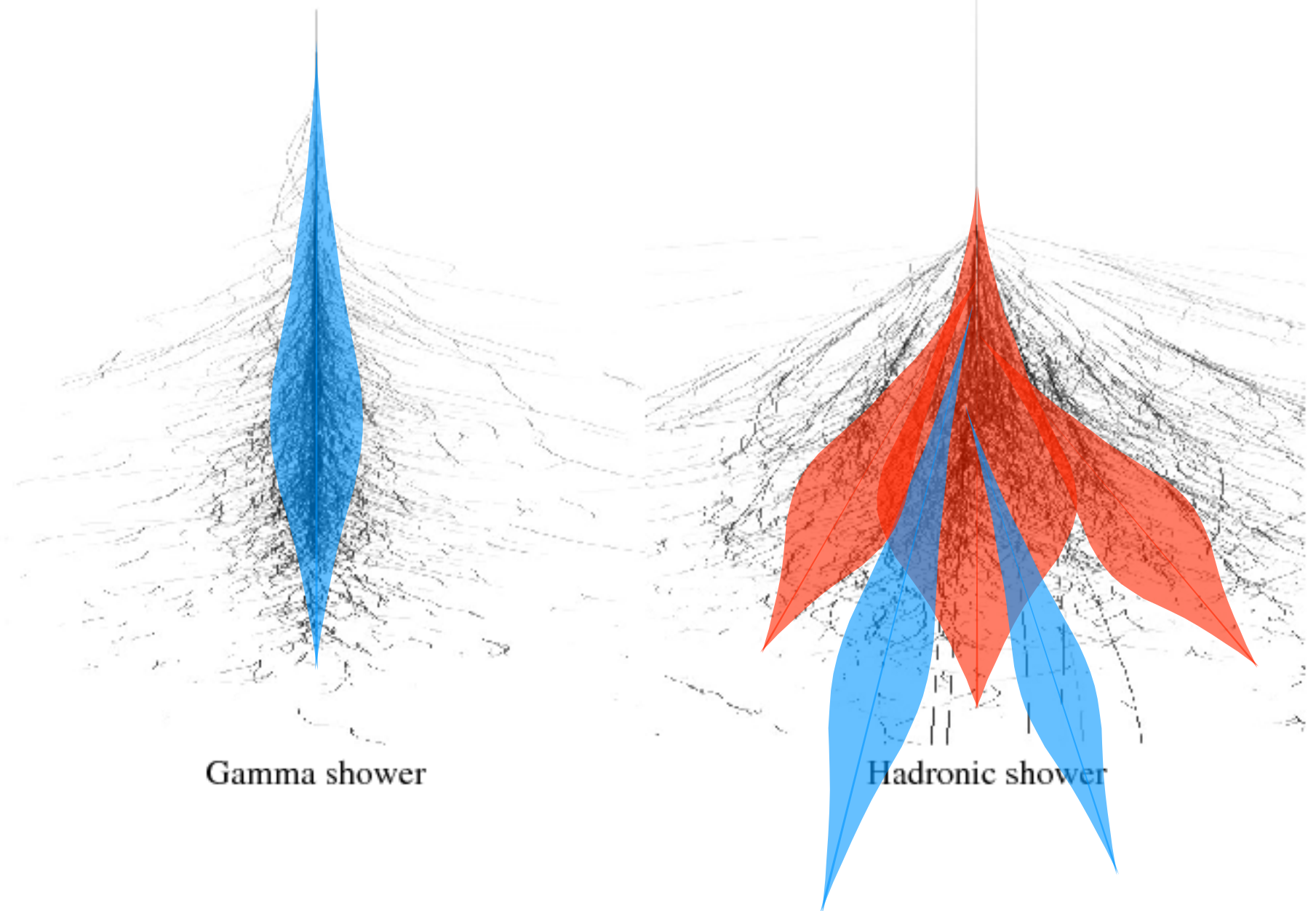
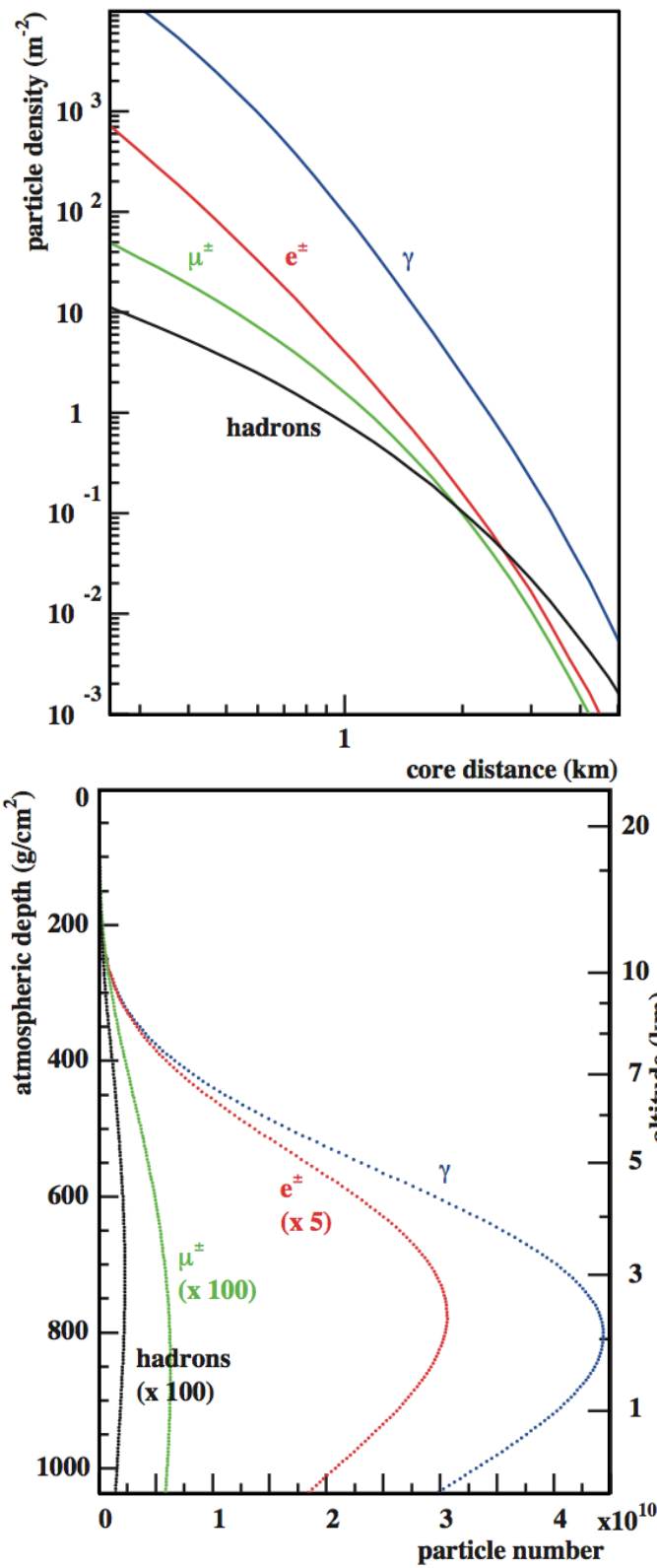
## EM and hadronic showers

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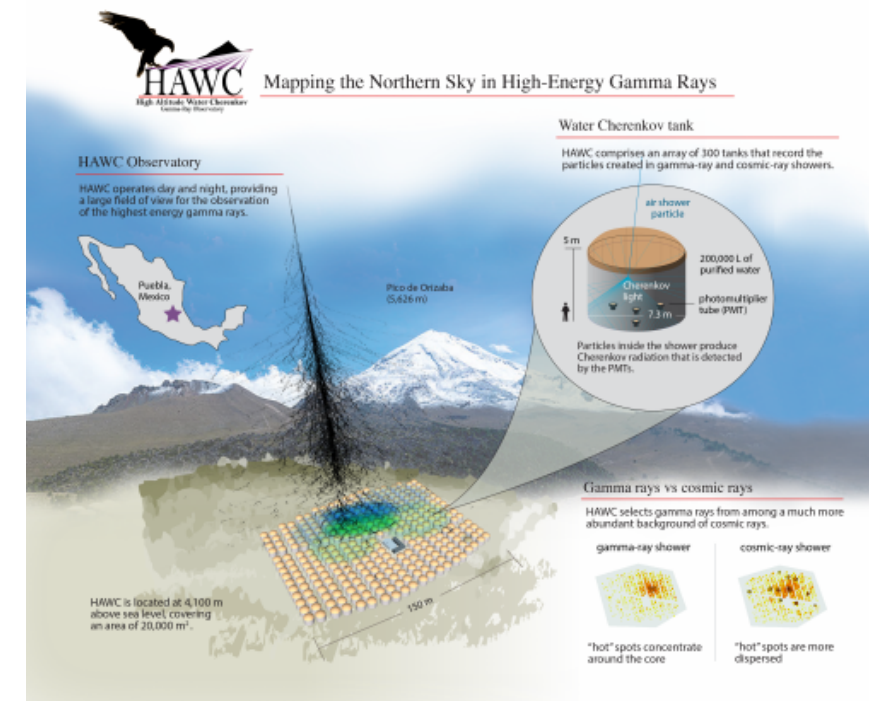




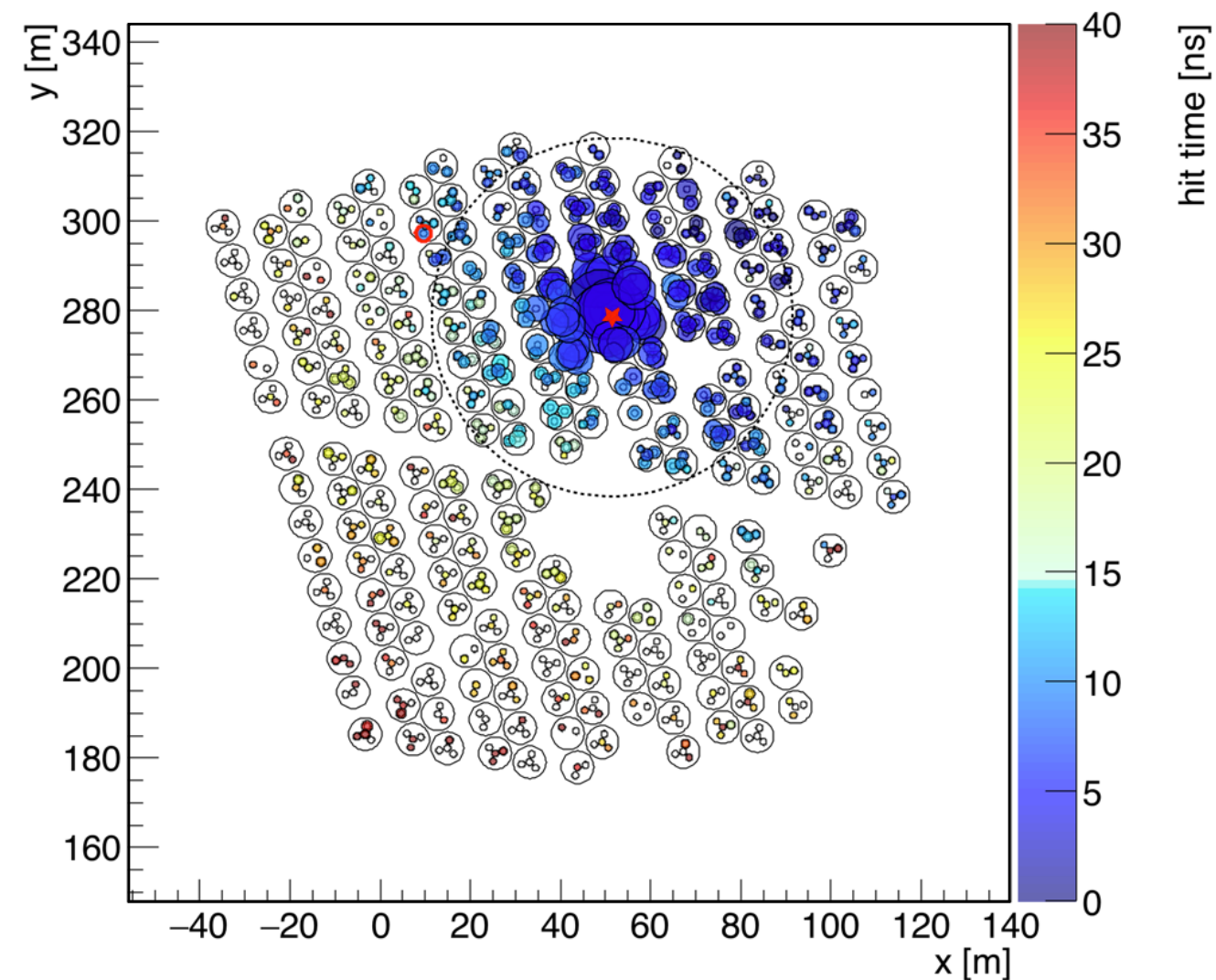
# extensive air showers

## experimental example: HAWC

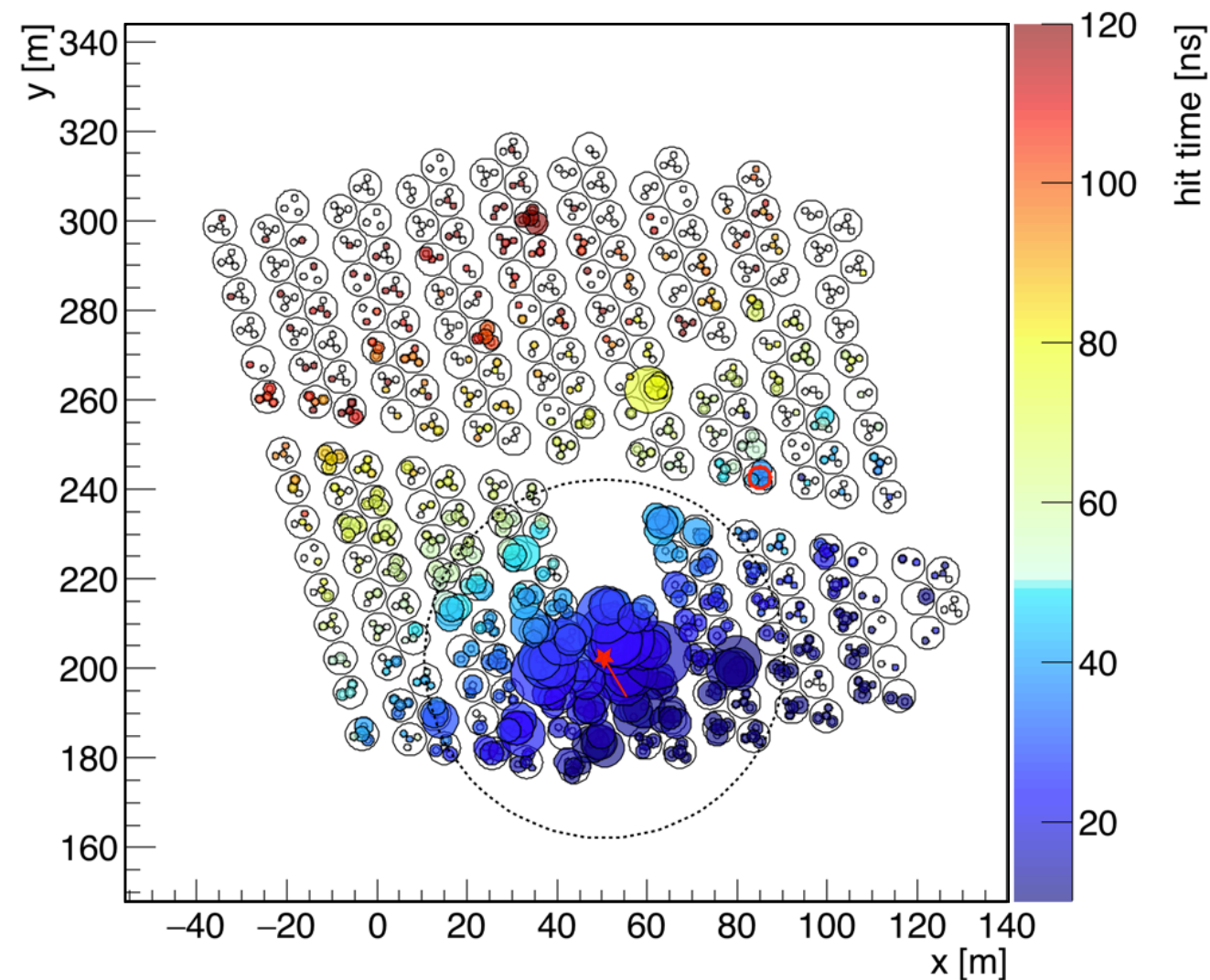
which event is induced by  
a cosmic ray **particle** and which by  
a **photon** ?



Run 2054, TS 584212, Ev# 226, CXPE40= 21.2, Cmptness= 28.3



Run 2118, TS 45004, Ev# 41, CXPE40= 55.7, Cmptness= 10.7



# extensive air showers

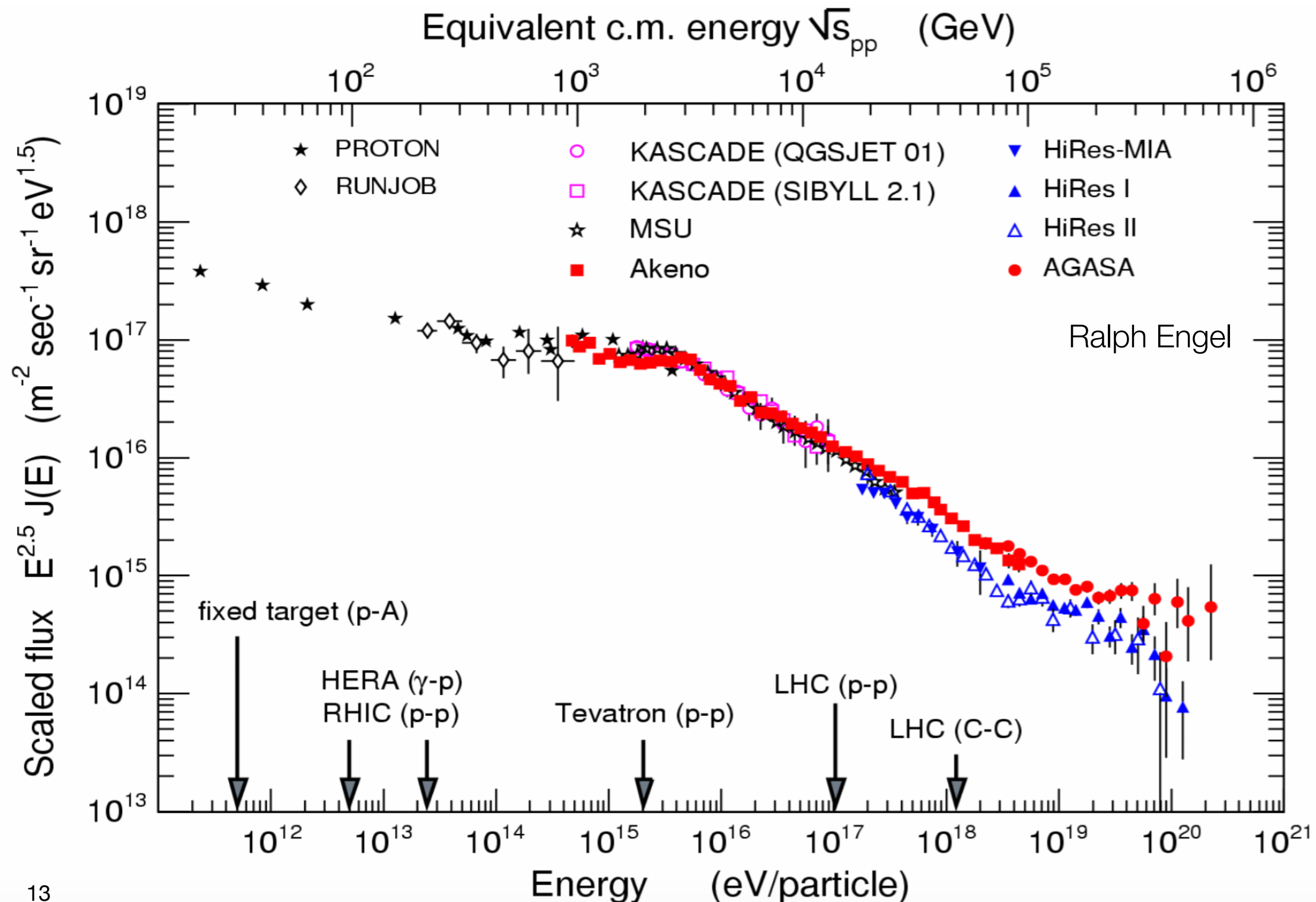
naturally and artificially generated energy

incident particles come

in different masses

with complicated spectra

largely uncertain at high energy



# extensive air showers

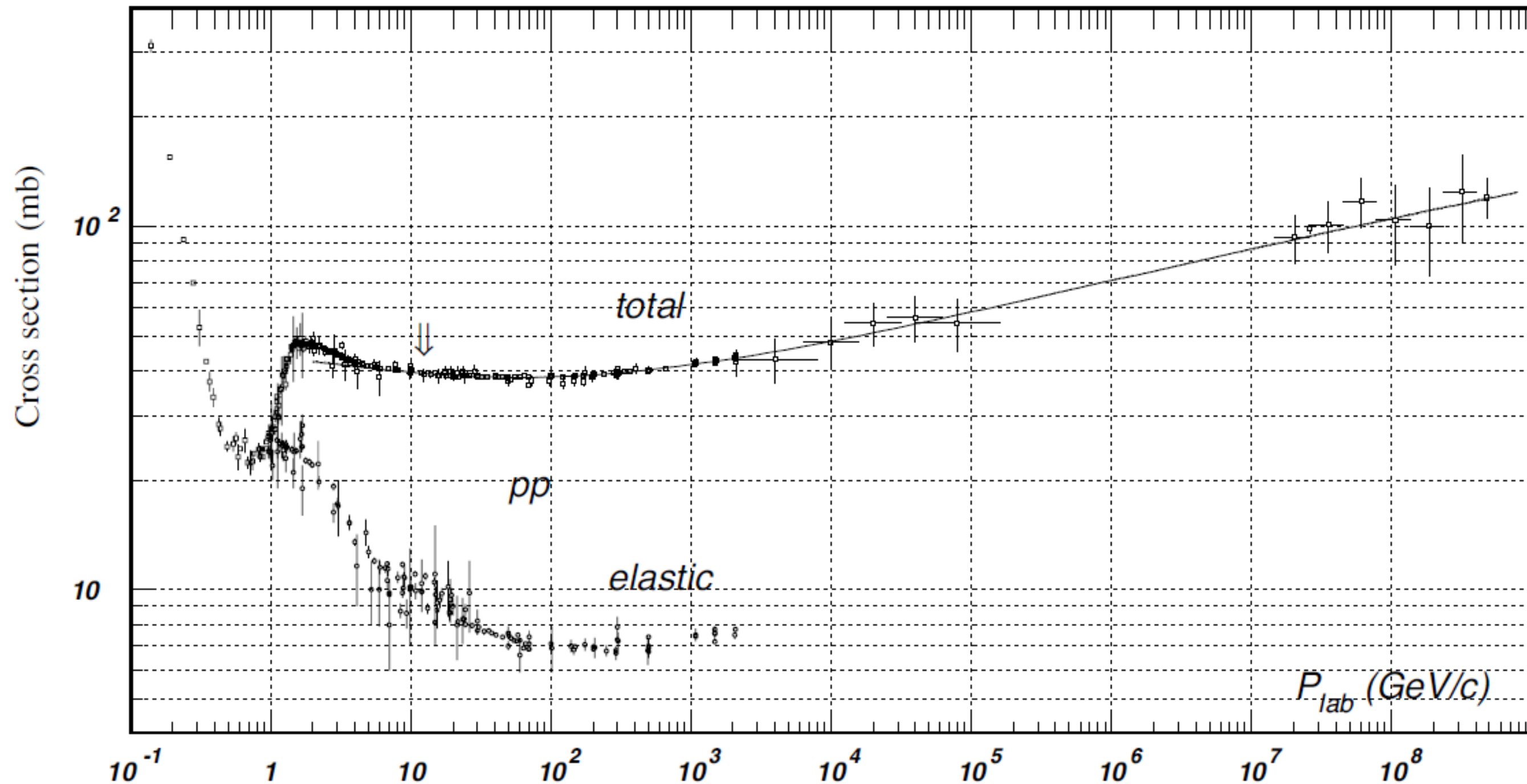
pp cross section

incident particles come

in different masses

with complicated spectra

largely uncertain at high energy



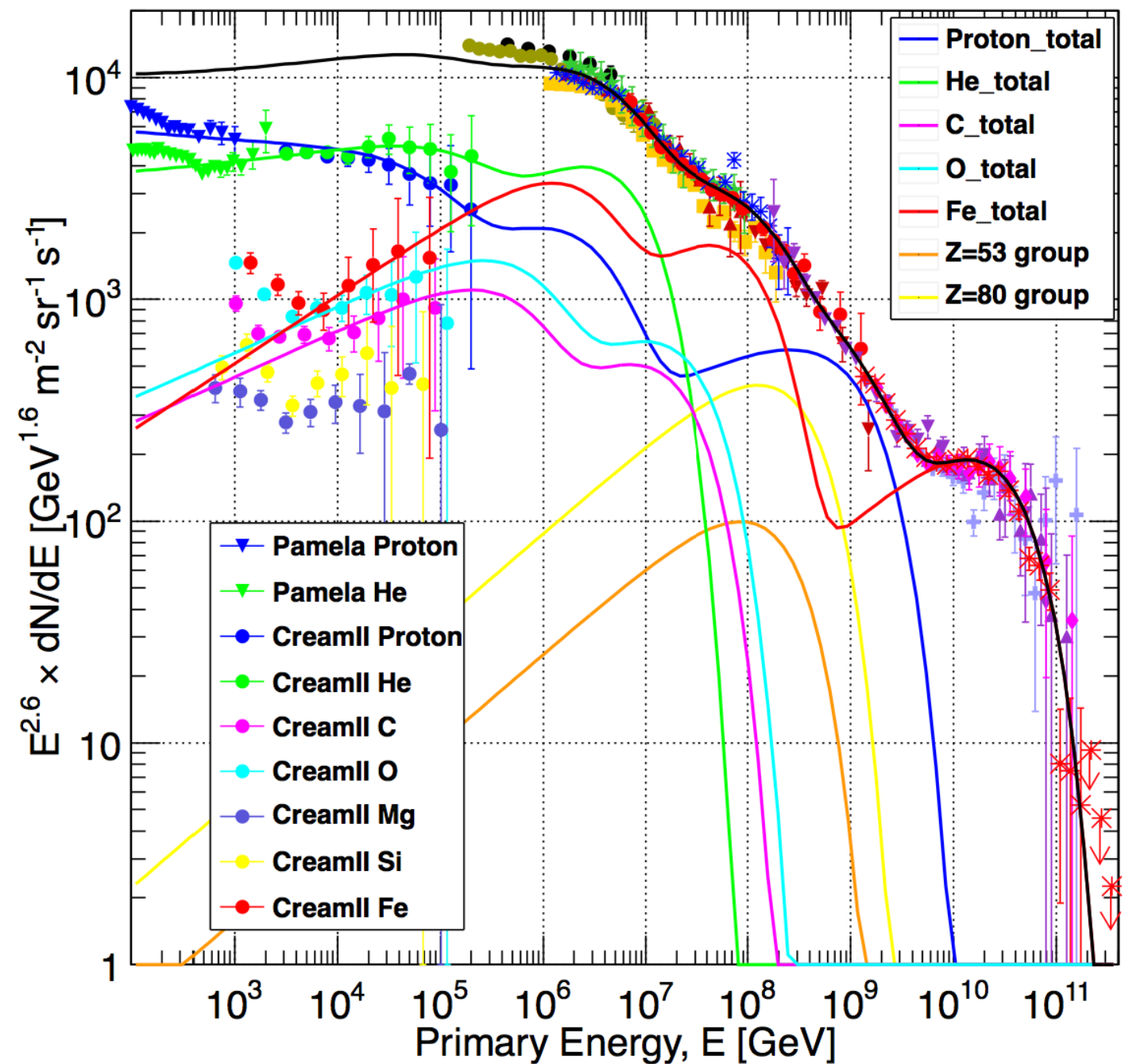


# extensive air showers

## EM and hadronic showers

incident particles come  
in different masses  
with complicated spectra  
largely uncertain at high energy

$$\begin{aligned}
 p + p \rightarrow & p(n) + p(n) \\
 & + \pi^+ + \pi^- + \pi^0 \\
 & + K^+ + K^- + K^0 + \bar{K}^0 + \dots \\
 & + p(n) + \bar{p}(\bar{n}) \\
 & + \text{charm} + \text{heavy quarks} + \dots \\
 & + e^\pm + \mu^\pm + \nu + \dots \\
 & + \dots
 \end{aligned}$$



# extensive air showers

## EM and hadronic showers

---

incident particles come  
in different masses  
with complicated spectra  
largely uncertain at high energy

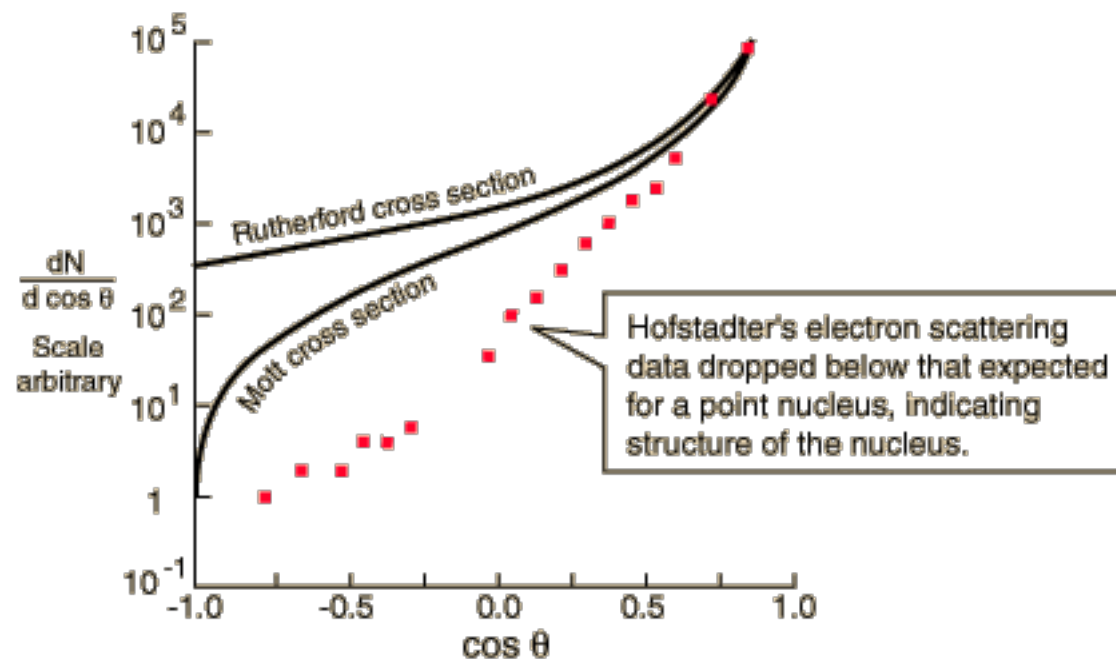
$$\begin{aligned} p + p \rightarrow & p(n) + p(n) \\ & + \pi^+ + \pi^- + \pi^0 \\ & + K^+ + K^- + K^0 + \bar{K}^0 + \dots \\ & + p(n) + \bar{p}(\bar{n}) \\ & + \text{charm} + \text{heavy quarks} + \dots \\ & + e^\pm + \mu^\pm + \nu + \dots \\ & + \dots \end{aligned}$$

leading baryons: 40-50% of energy  
pions: the most abundant particles  
kaons: *strange* particles  
baryon/anti-baryon pairs  
heavy quark production  
leptons  
other mesons and baryons

# Quantum Chromo-Dynamics

very quick overview ... via 4 Nobel Prizes

- “for his *pioneering studies of electron scattering in atomic nuclei and for his consequent discoveries concerning the structure of nucleons*”



- atomic nuclei have a finite size

## ROBERT HOFSTADTER



The Nobel Prize in Physics 1961

Robert Hofstadter, Rudolf Mössbauer

## Robert Hofstadter - Biographical



Robert Hofstadter, Professor of Physics at Stanford University, was born in New York, N.Y., of parents Louis Hofstadter and Henrietta Koenigsberg, on February 5, 1915.

Hofstadter attended elementary and high schools in New York City, and was graduated in 1935 from the College of the City of New York with the B.S. degree, *magna cum laude*.

On graduation from college Hofstadter received the Kenyon Prize in Mathematics and Physics, and a little later the Coffin Fellowship, awarded by the General Electric Company. He went to graduate school at Princeton University where he studied physics from 1935 - 1938, and received both the M.A. and Ph.D. degrees in 1938 from that institution. His Ph.D. work was concerned with

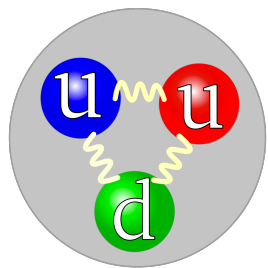


# Quantum Chromo-Dynamics

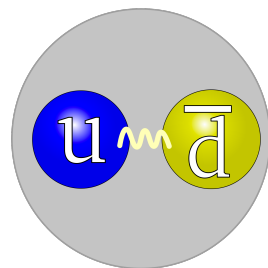
very quick overview ... via 4 Nobel Prizes

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- “for his contributions and discoveries concerning the classification of elementary particles and their interactions”
- introduced the **quark** - constituent of hadrons - independently from George Zweig



baryons



mesons

## MURRAY GELL-MAN



The Nobel Prize in Physics 1969  
Murray Gell-Mann

## The Nobel Prize in Physics 1969



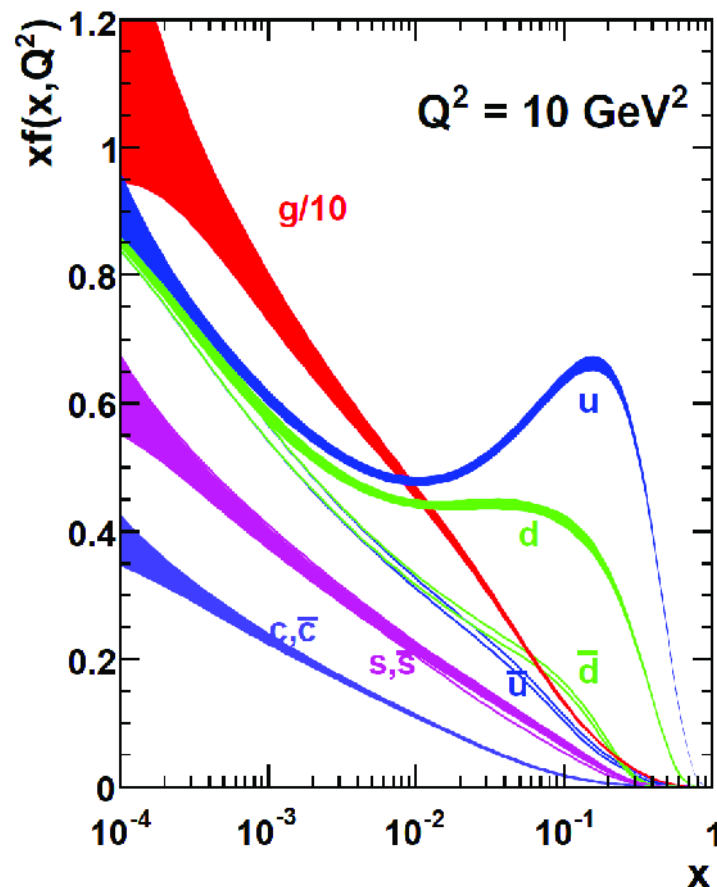
Murray Gell-Mann  
Prize share: 1/1

The Nobel Prize in Physics 1969 was awarded to Murray Gell-Mann  
*"for his contributions and discoveries concerning the classification  
of elementary particles and their interactions".*

# Quantum Chromo-Dynamics

very quick overview ... via 4 Nobel Prizes

- “for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics”



parton  
distribution functions

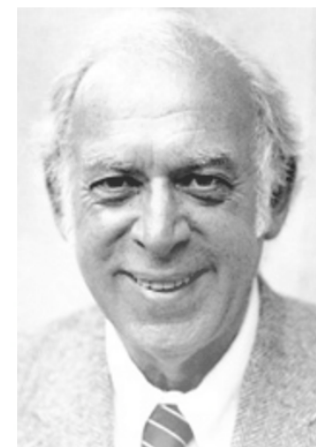
## FRIEDMAN, KENDALL & TAYLOR



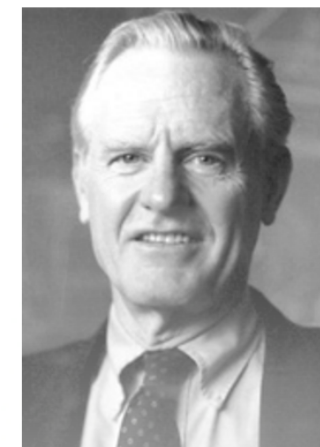
The Nobel Prize in Physics 1990

Jerome I. Friedman, Henry W. Kendall, Richard E. Taylor

## The Nobel Prize in Physics 1990



Jerome I. Friedman  
Prize share: 1/3



Henry W. Kendall  
Prize share: 1/3



Photo: T. Nakashima  
Richard E. Taylor  
Prize share: 1/3

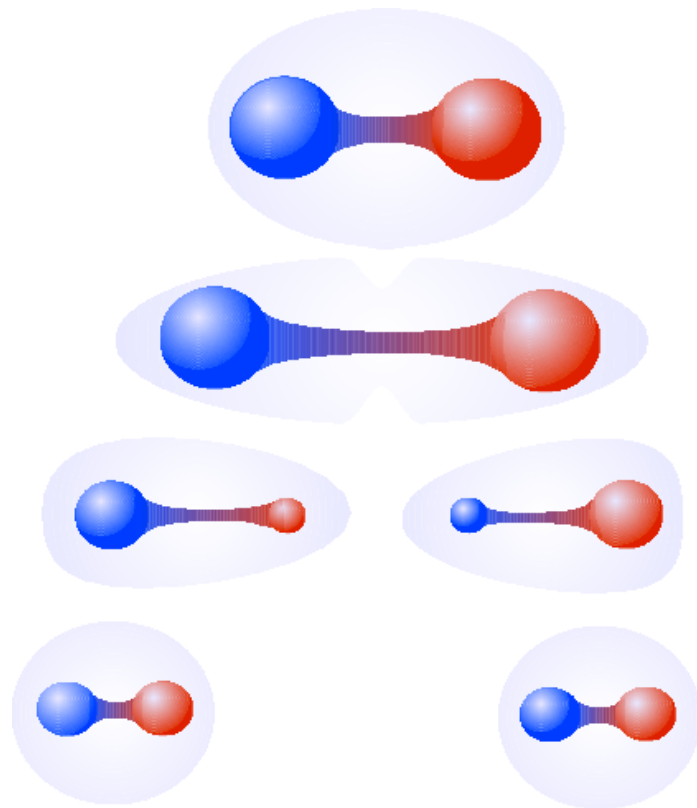
The Nobel Prize in Physics 1990 was awarded jointly to Jerome I. Friedman, Henry W. Kendall and Richard E. Taylor “for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of

# Quantum Chromo-Dynamics

very quick overview ... via 4 Nobel Prizes

---

- “for the discovery of asymptotic freedom in the theory of the strong interaction”
- color *charge* of quarks & confinement



## GROSS, POLITZER & WILCZEK



The Nobel Prize in Physics 2004

David J. Gross, H. David Politzer, Frank Wilczek

## The Nobel Prize in Physics 2004



David J. Gross

Prize share: 1/3



H. David Politzer

Prize share: 1/3



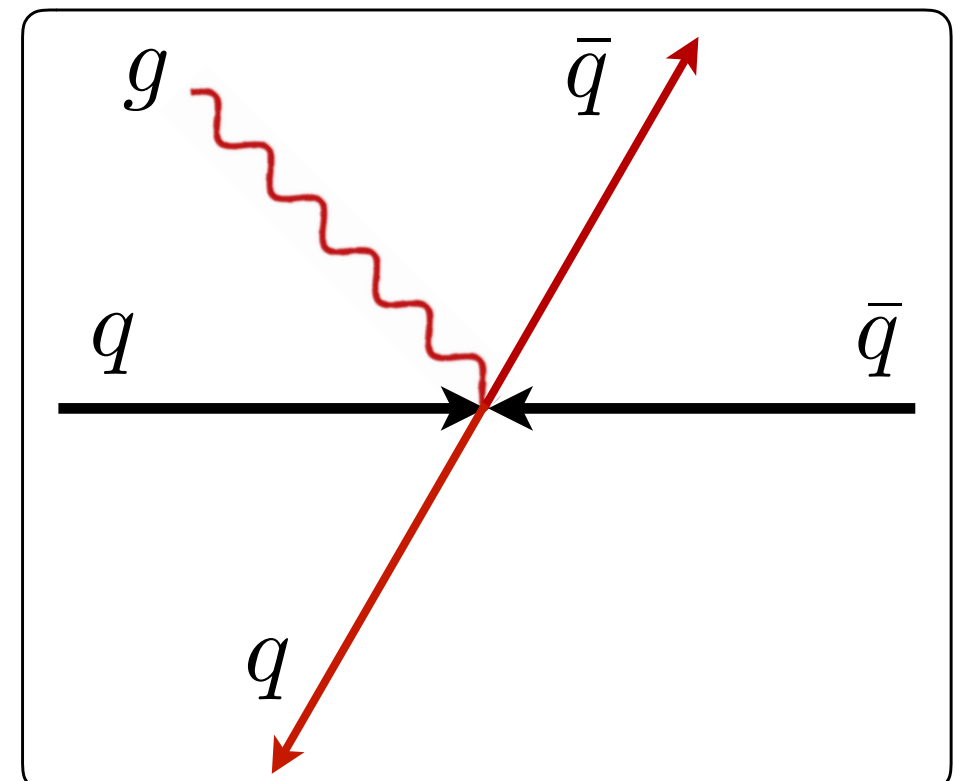
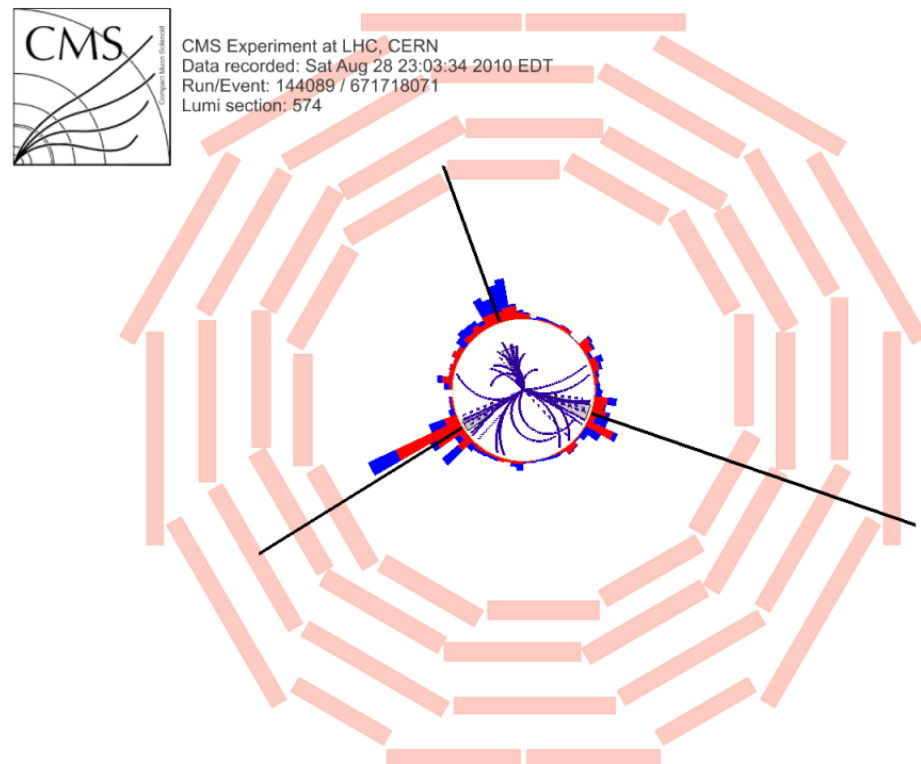
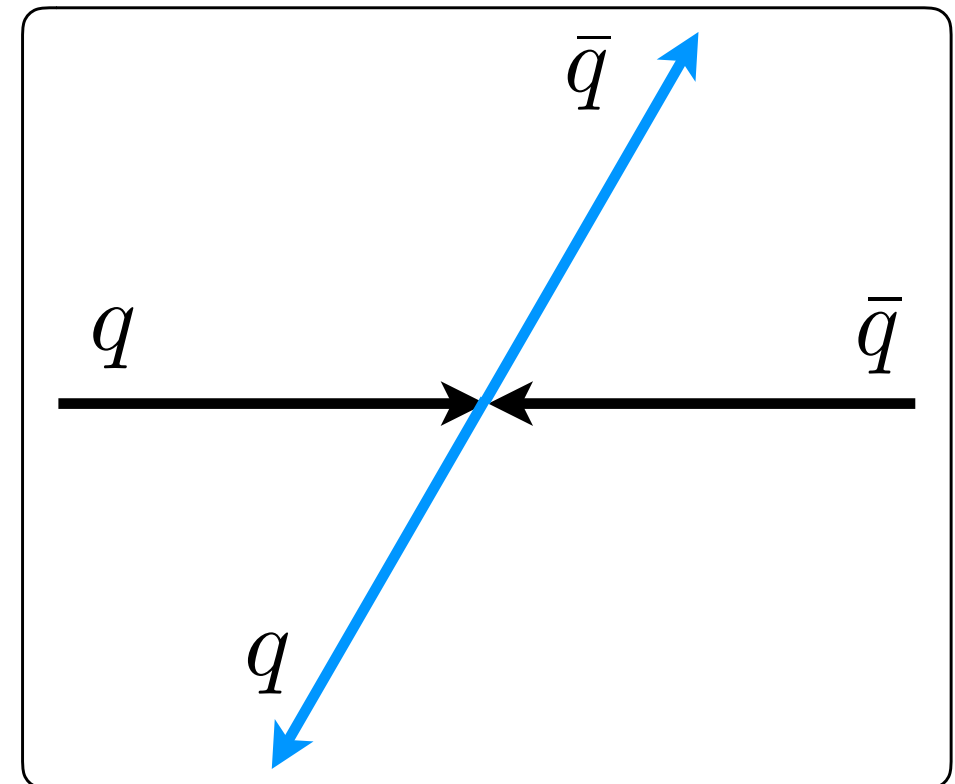
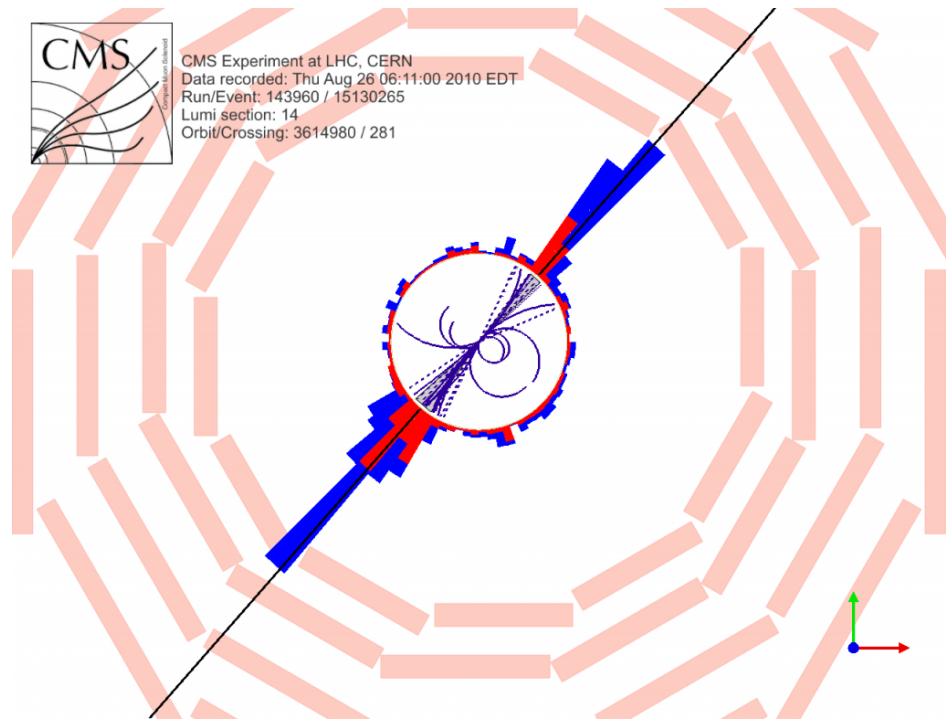
Frank Wilczek

Prize share: 1/3

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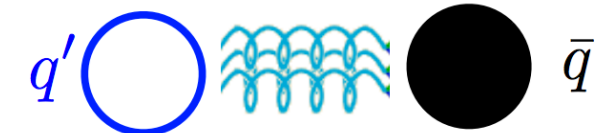
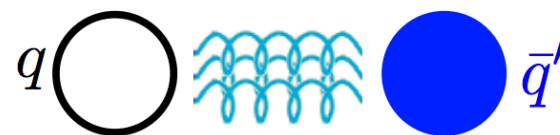
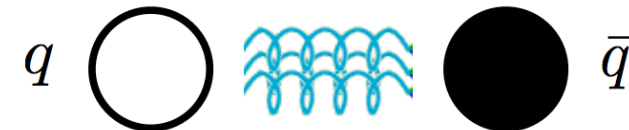
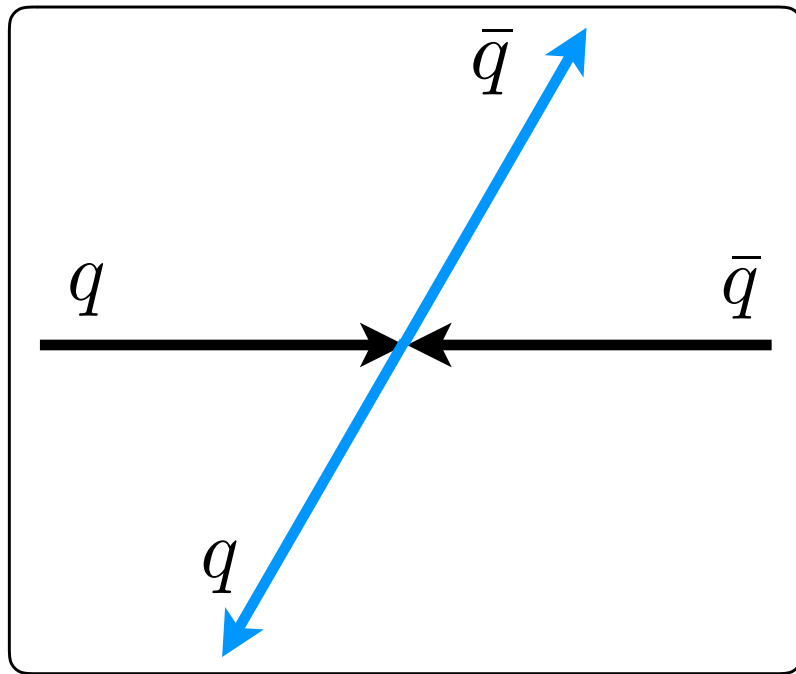
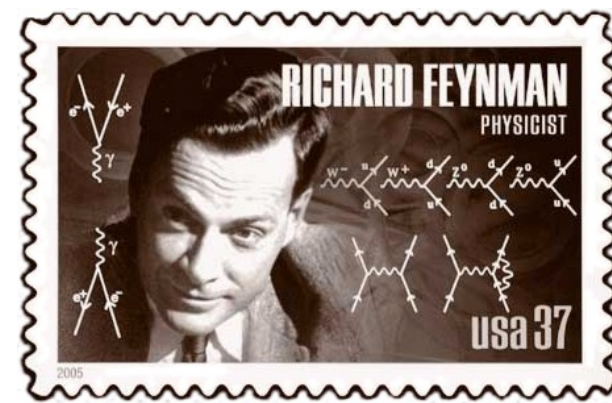
# hadronization

## from partons to particles

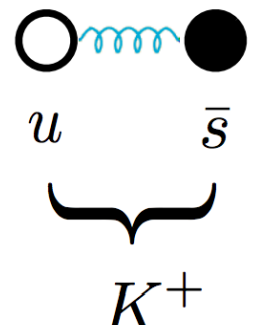
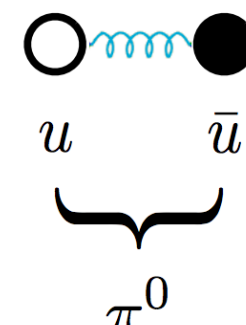
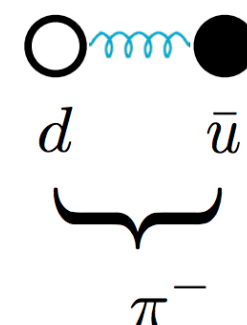
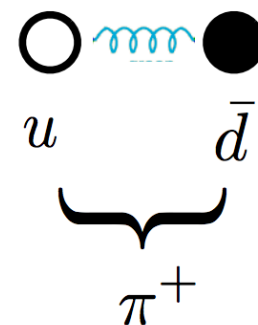
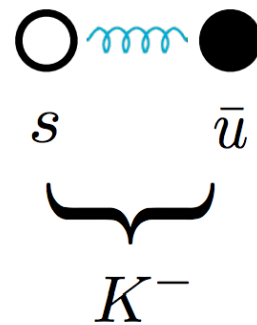


# hadronization

## from partons to particles



effects of  
confinement and  
asymptotic freedom

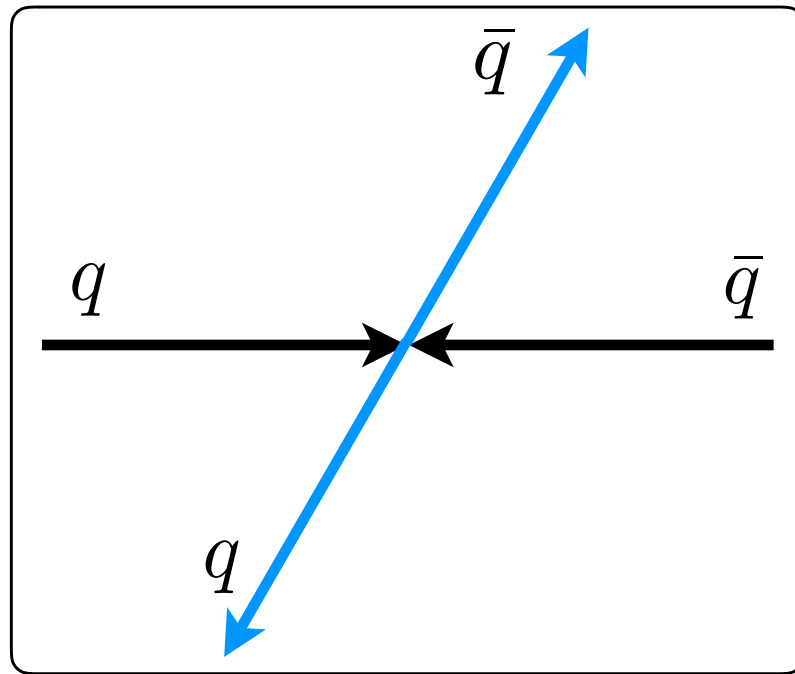
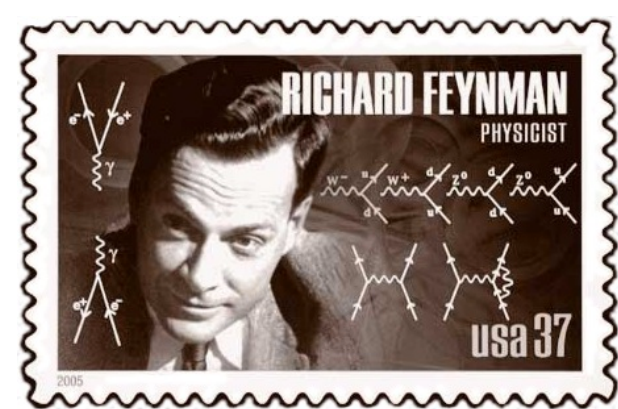


mesons

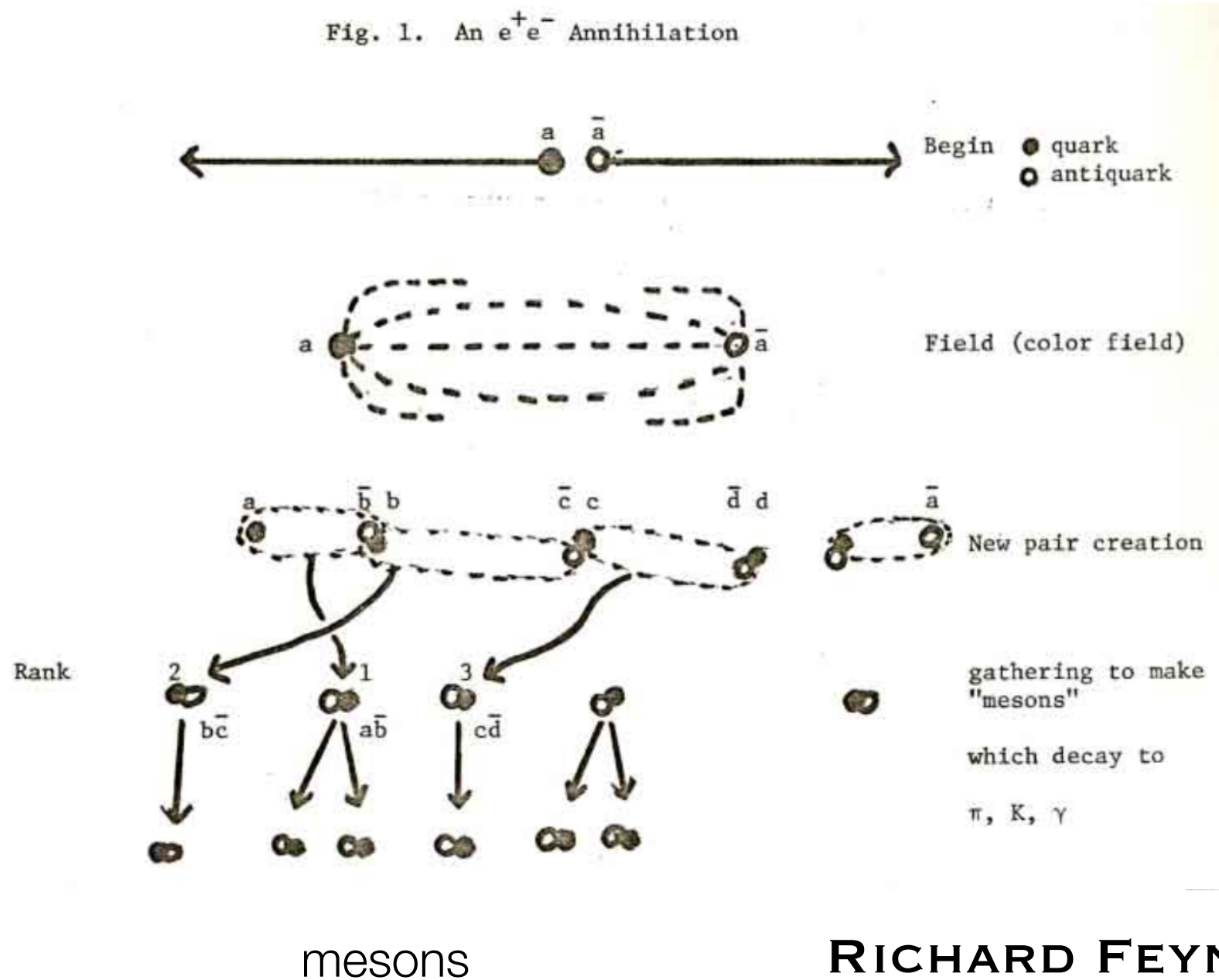


# hadronization

## from partons to particles



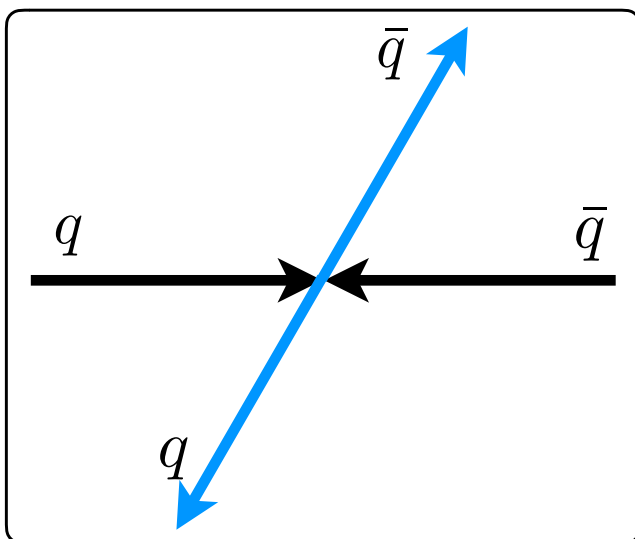
effects of  
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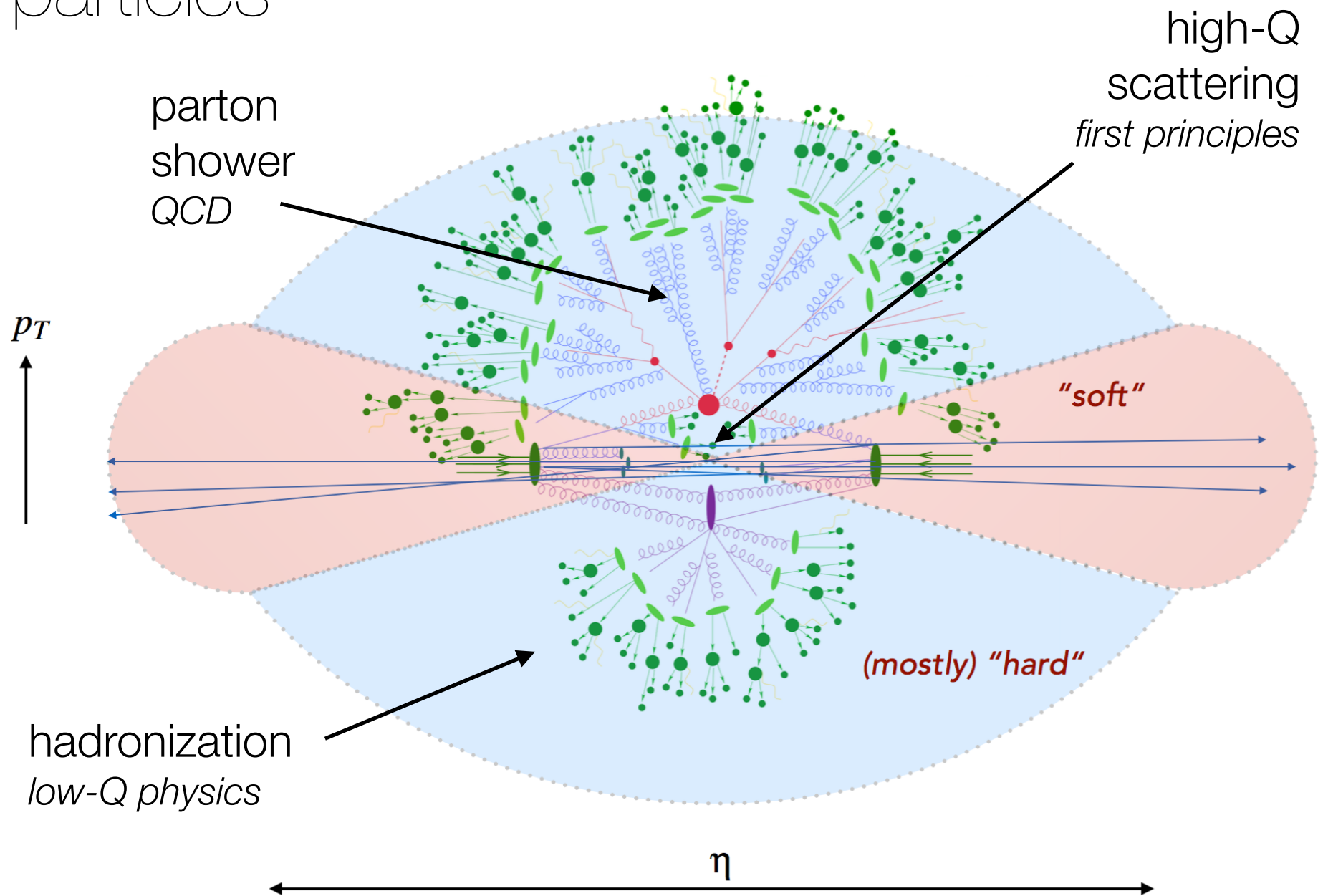
**RICHARD FEYNMAN**

# hadronization

## from partons to particles



high-Q  
scattering



hadronic interaction cross section  
multiplicity of secondary particles  
**must be obtained from collider experiments**

# Lorentz covariance

## rapidity & pseudo-rapidity

---

- Lorentz transformations are a hyperbolic rotation of space-time

rapidity  $\phi = \tanh^{-1} \left( \frac{|\vec{p}|}{E} \right)$   $\phi = \frac{1}{2} \ln \left( \frac{E + |\vec{p}|}{E - |\vec{p}|} \right)$

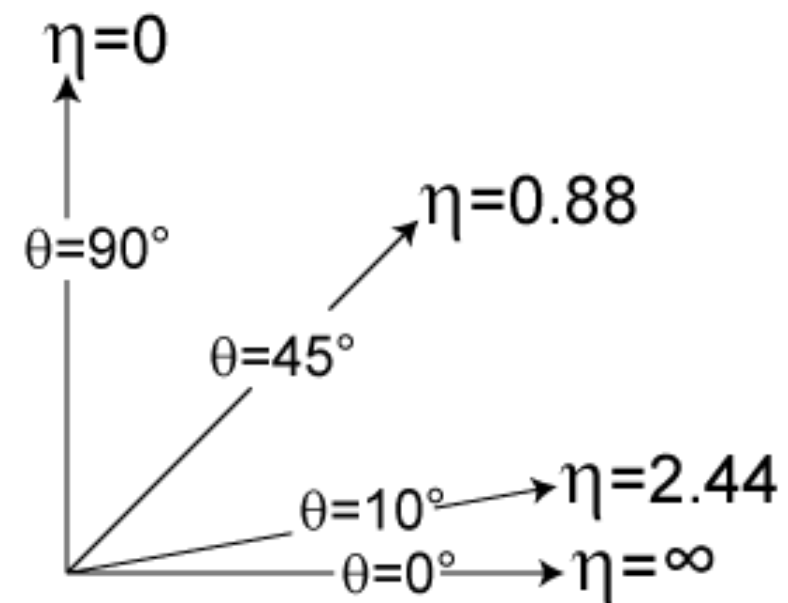
- in accelerator physics

$$y = \frac{1}{2} \ln \left( \frac{E + p_{\parallel}}{E - p_{\parallel}} \right)$$

- at ultra-relativistic limit ( $m \sim 0$ )

pseudo-rapidity  $\eta = \frac{1}{2} \ln \left( \frac{|\vec{p}| + p_{\parallel}}{|\vec{p}| - p_{\parallel}} \right)$

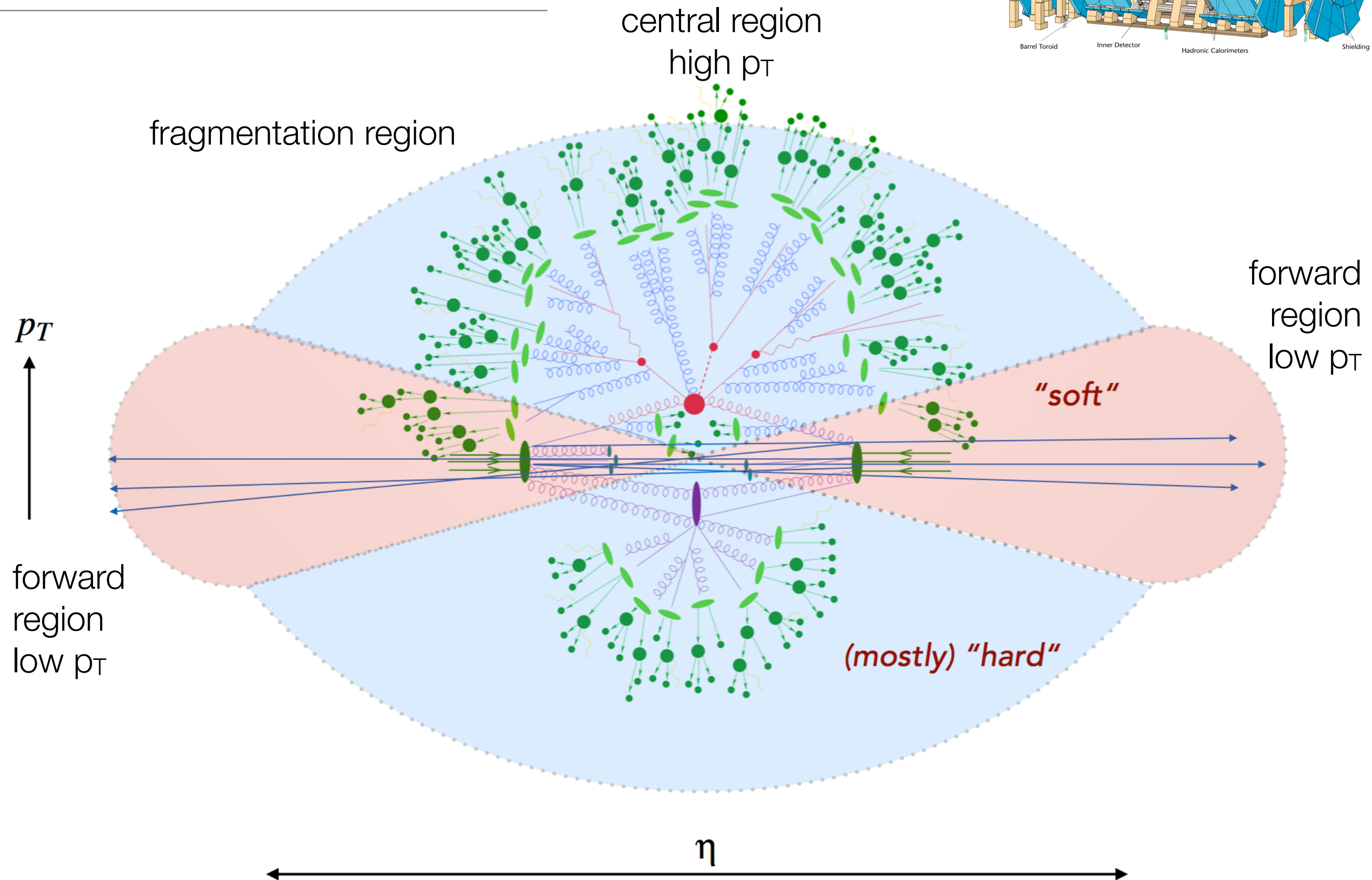
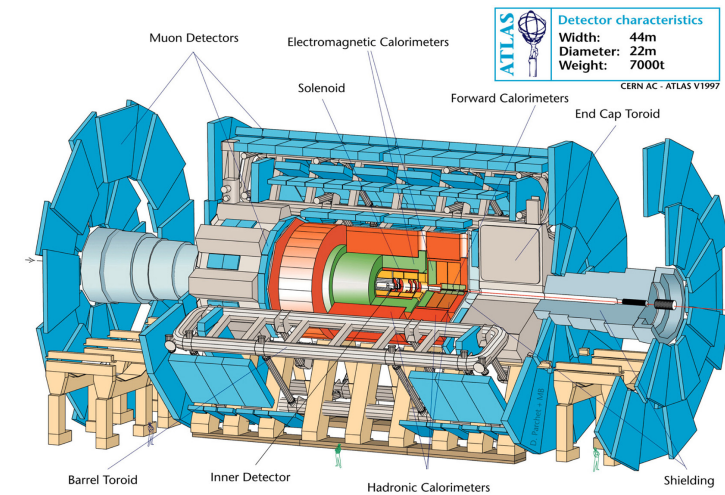
$$\eta = -\ln \left( \tan \frac{\theta}{2} \right)$$





# hadronization

## high energy particle interactions

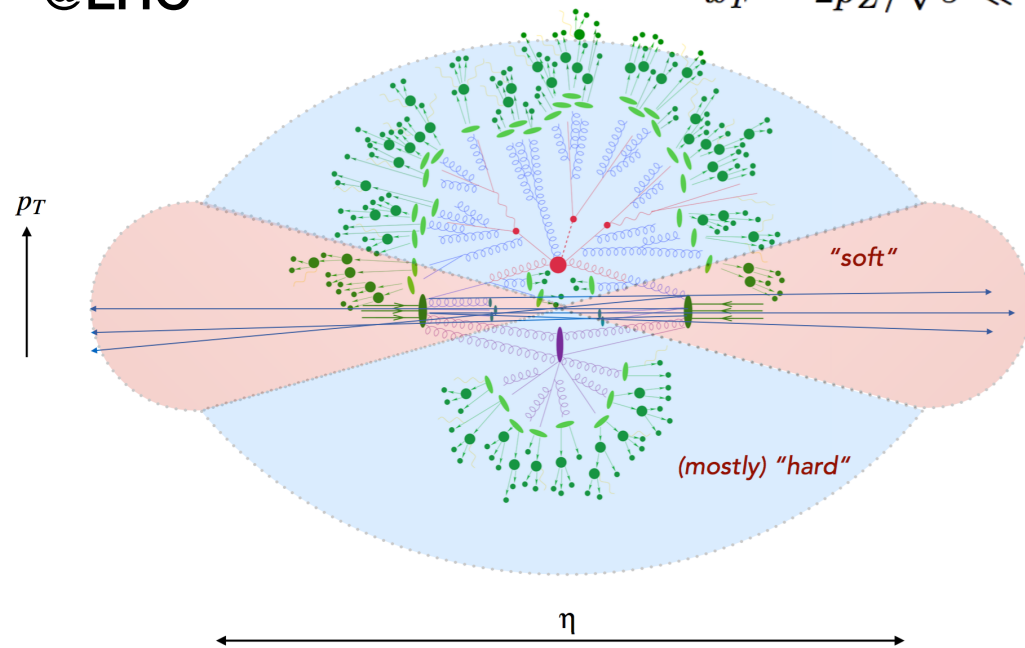


# hadronization

## high energy particle interactions

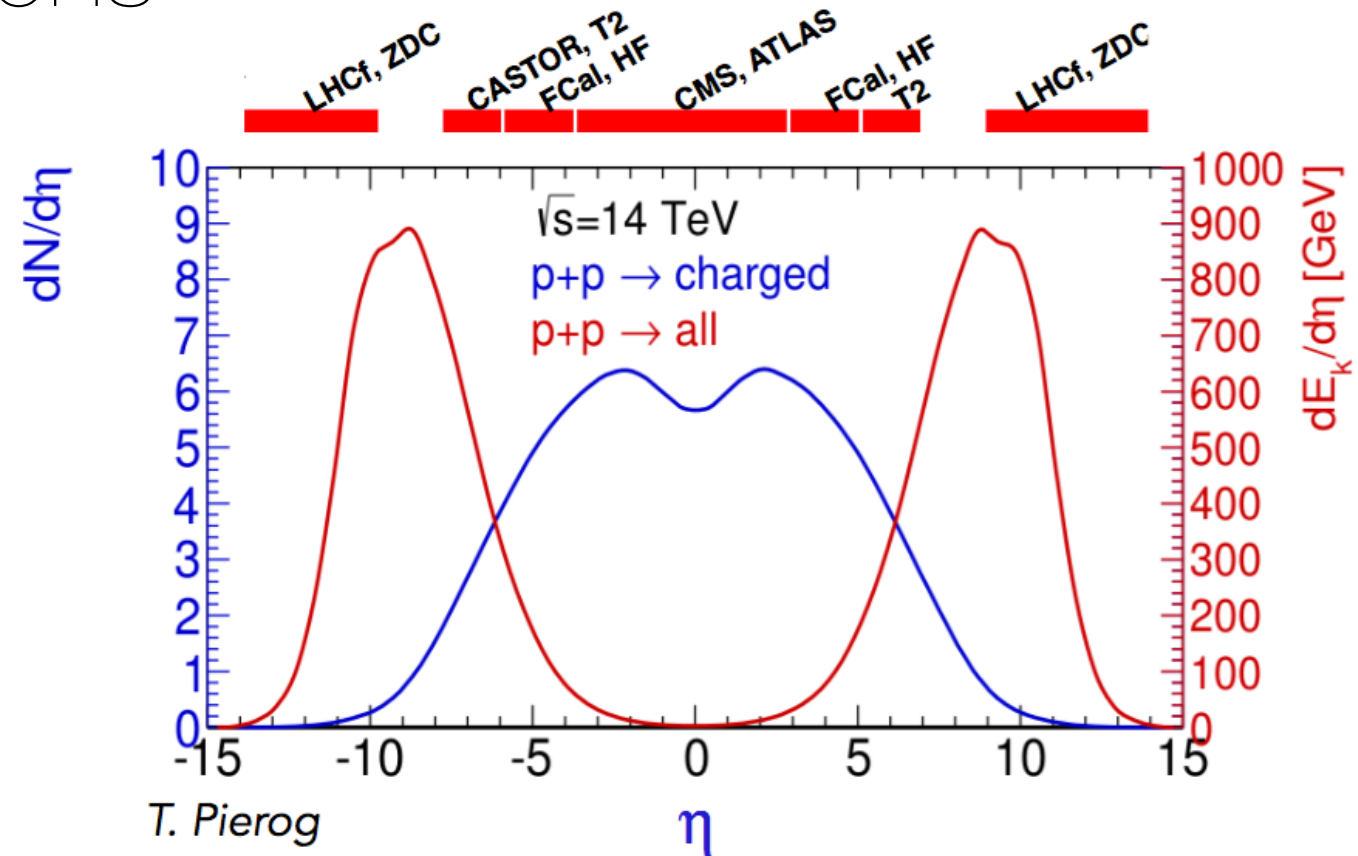
@LHC

$$x_F = 2p_Z/\sqrt{s} \ll 0.1$$



### hard (central region)

- high particle number density
- low energy density
- heavy particles decay in this region
- observed by **collider experiments**



### soft (forward region)

- low particle number density
- high energy density
- product of valence quark interactions
- crucial in **cosmic ray physics**

# hadronization

## high energy particle interactions

---

### hard (central region)

- high particle number density
- low energy density
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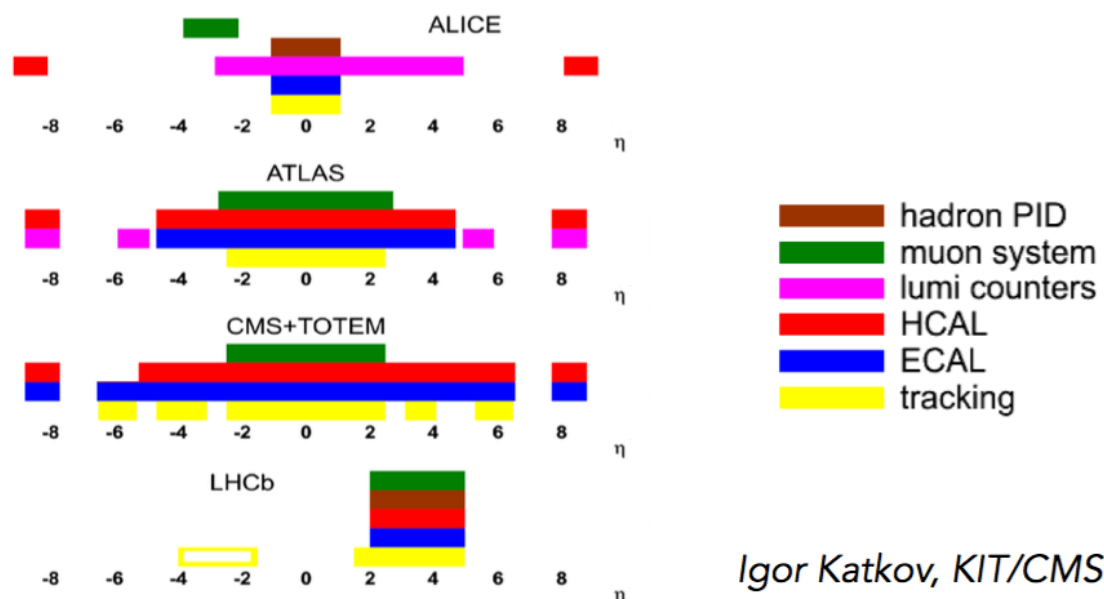
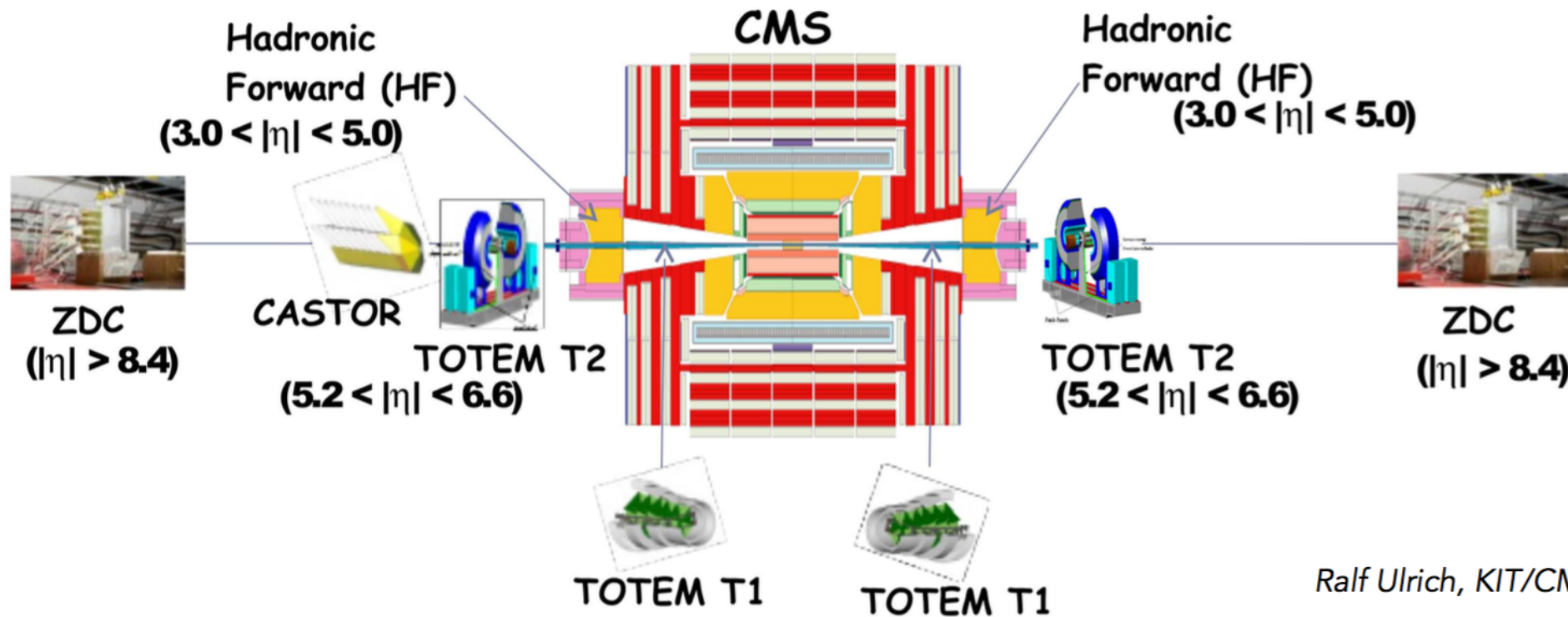
### soft (forward region)

- low particle number density
- high energy density
- product of valence quark interactions
- crucial in **cosmic ray physics**

- ▶ QCD can be calculated only in **perturbative** limit (pQCD or hard QCD) when strong coupling constant is **small** (at high  $p_T$ )
- ▶ no calculable theory in diffractive **non-perturbative** (soft QCD) limit (at low  $p_T$ )
- ▶ Gribov-**Regge** theory (pomeron *color-singlet* exchange) successfully applied

# particle detection in colliders

## Experimental acceptance

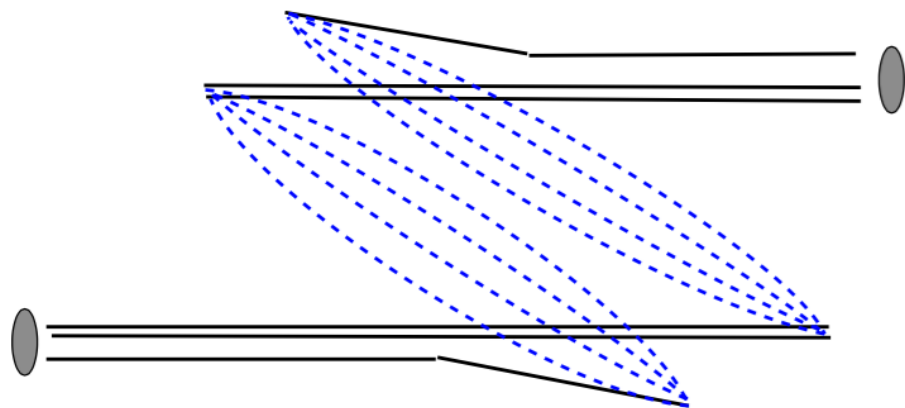


Pseudorapidity

$$\eta = -\ln \left( \tan \frac{\theta}{2} \right)$$

# hadronization

## two string model

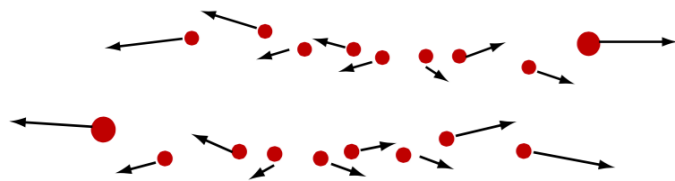
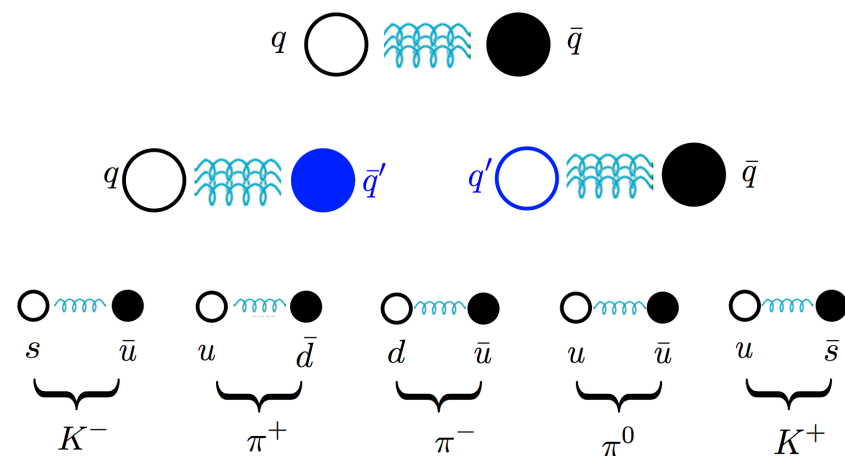


- ▶ the basic structure of a non diffractive pp interaction is made of **two strings** (colored partons). Flat rapidity  $dn/dy$  distributions.

- ▶ leading particle effects

- ▶ Feynman **scaling**:  $dn/dy$  independent of energy. (approximately valid in forward fragmentation region)

- ▶ distributions independent of energy

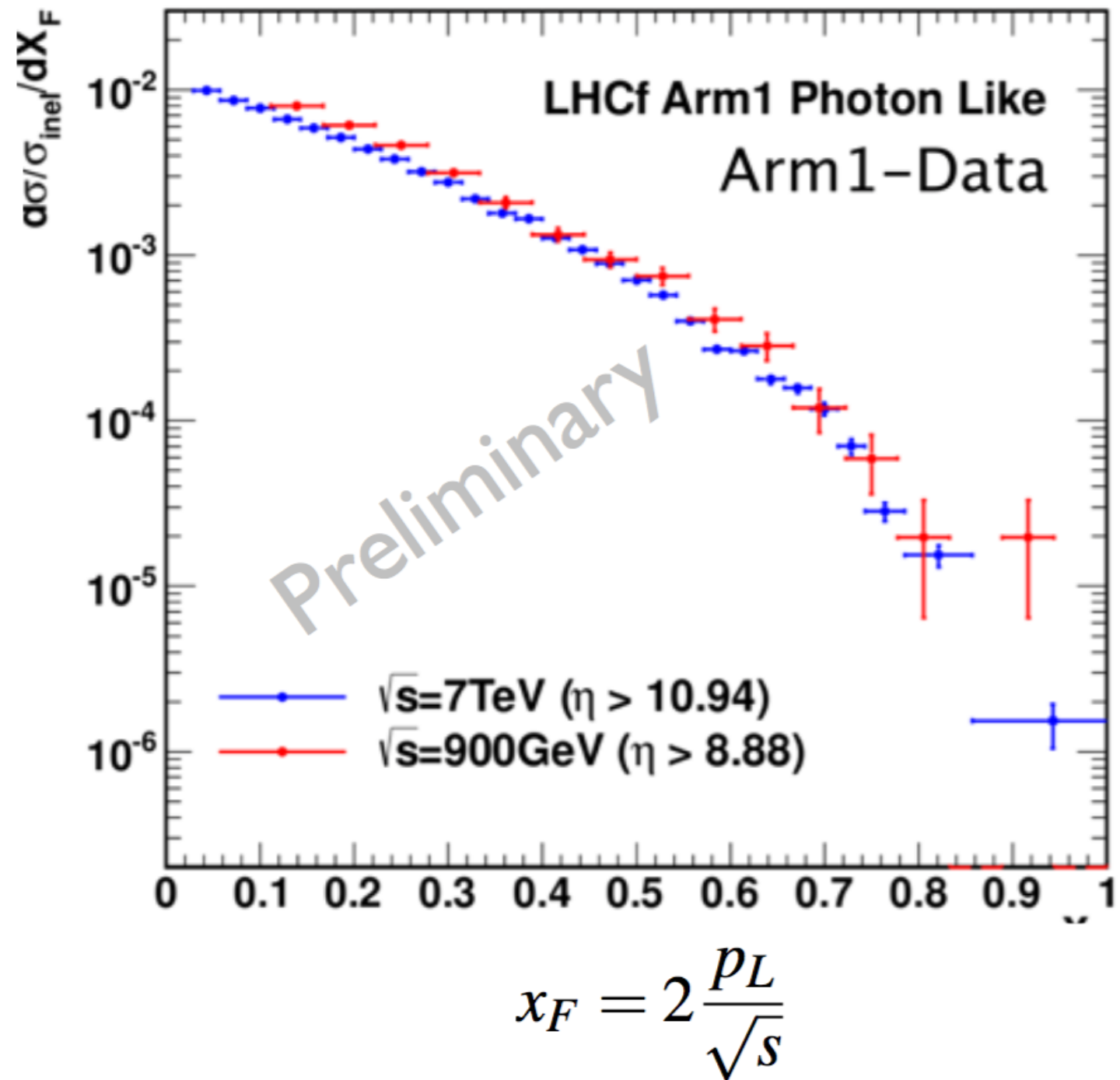
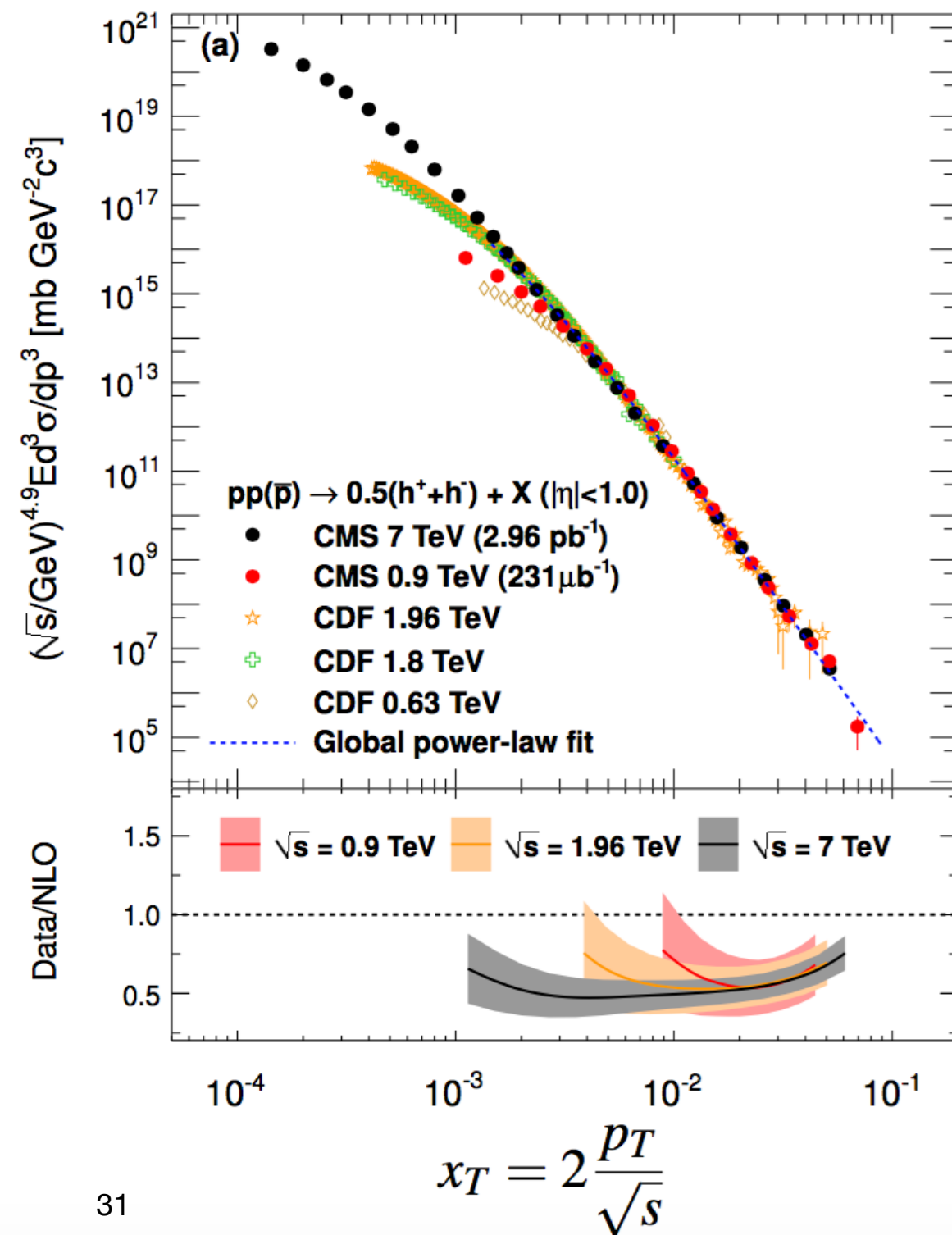


$$\frac{dN}{dx} \approx f(x) \quad x = E/E_{\text{prim}}$$



# hadronization

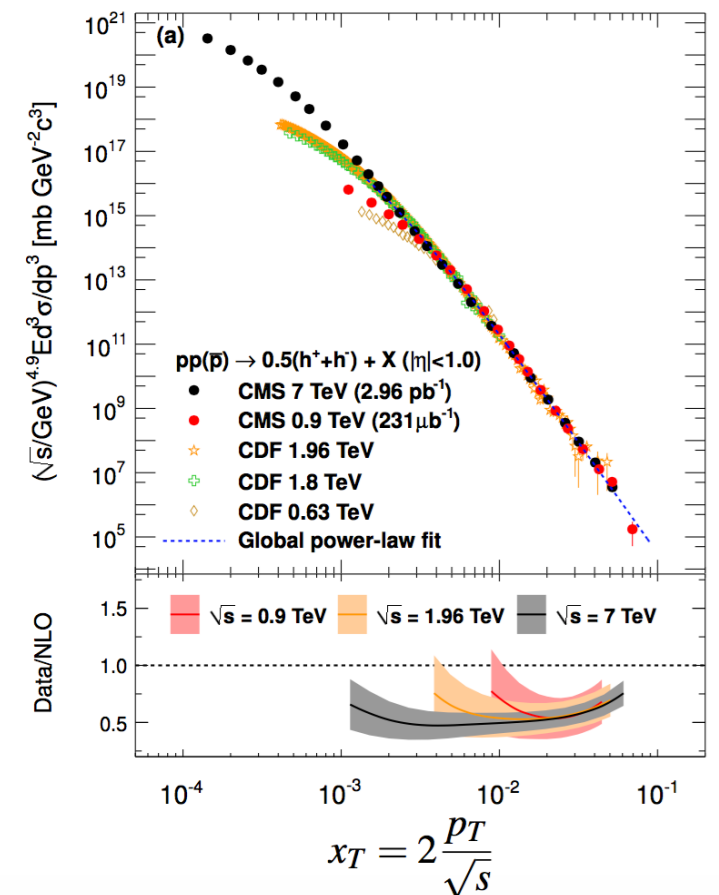
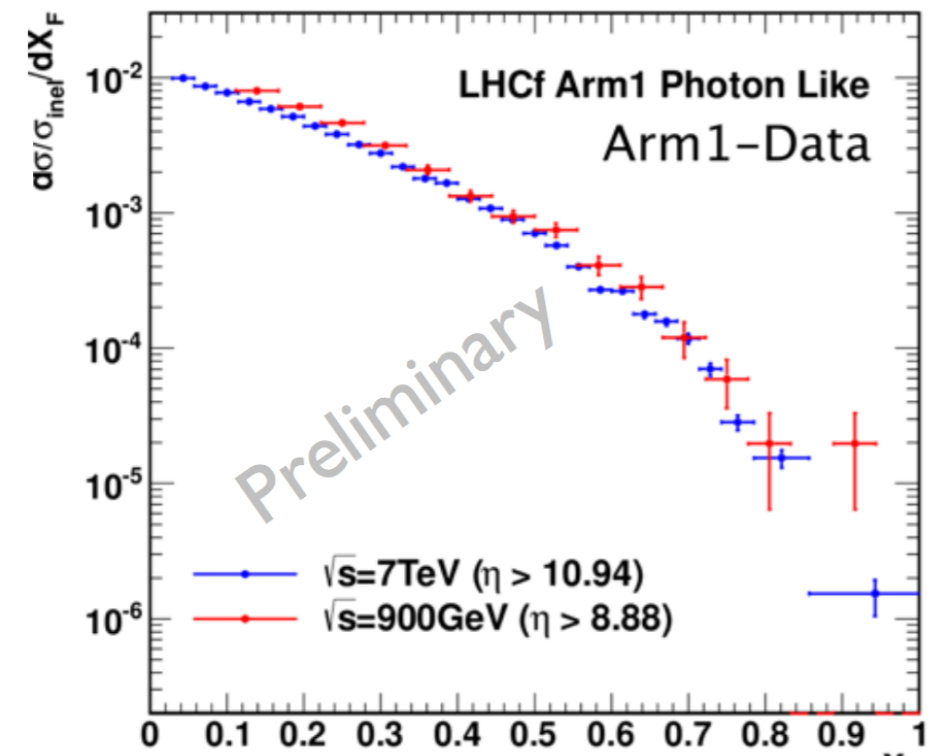
## experimental evidence of scaling



# hadronization

## experimental evidence of scaling

- ▶ cosmic rays indicate approximate scaling
- ▶ if scaling is perfect measurements at low energy would be sufficient for predictions at high energy
- ▶ in deep inelastic scattering at high momentum partons are *free* and collision with one parton is not affected by others: **scaling!**
- ▶ scaling in fractional momentum  $x$  is violated for low values of  $x$ . Partons are not free: **scaling violation!**
- ▶  $p = (uud) + (gqq \text{ sea})$



# hadronic interaction models

bridging particle physics to cosmic ray interactions

---

## High energy models:

DPMJET II.5 and III (Ranft / Roesler, RE & Ranft)

neXus 2.0 and 3.0 (Drescher, Hladik, Ostapchenko, Pierog & Werner)

QGSJET 98 and 01 (Kalmykov & Ostapchenko)

SIBYLL 1.7 and 2.1 (Engel / RE, Fletcher, Gaisser, Lipari & Stanev)

- Gribov-Regge type models, minijets
- Parametrizations of data

## Low/intermediate energy models:

GHEISHA (Fesefeldt)

Hillas' splitting algorithm (Hillas)

FLUKA (Fasso, Ferrari, Ranft & Sala)

UrQMD (Bass, Bleicher et al.)

TARGET (RE, Gaisser, Protheroe & Stanev)

HADRIN/NUCRIN (Hänßgen & Ranft)

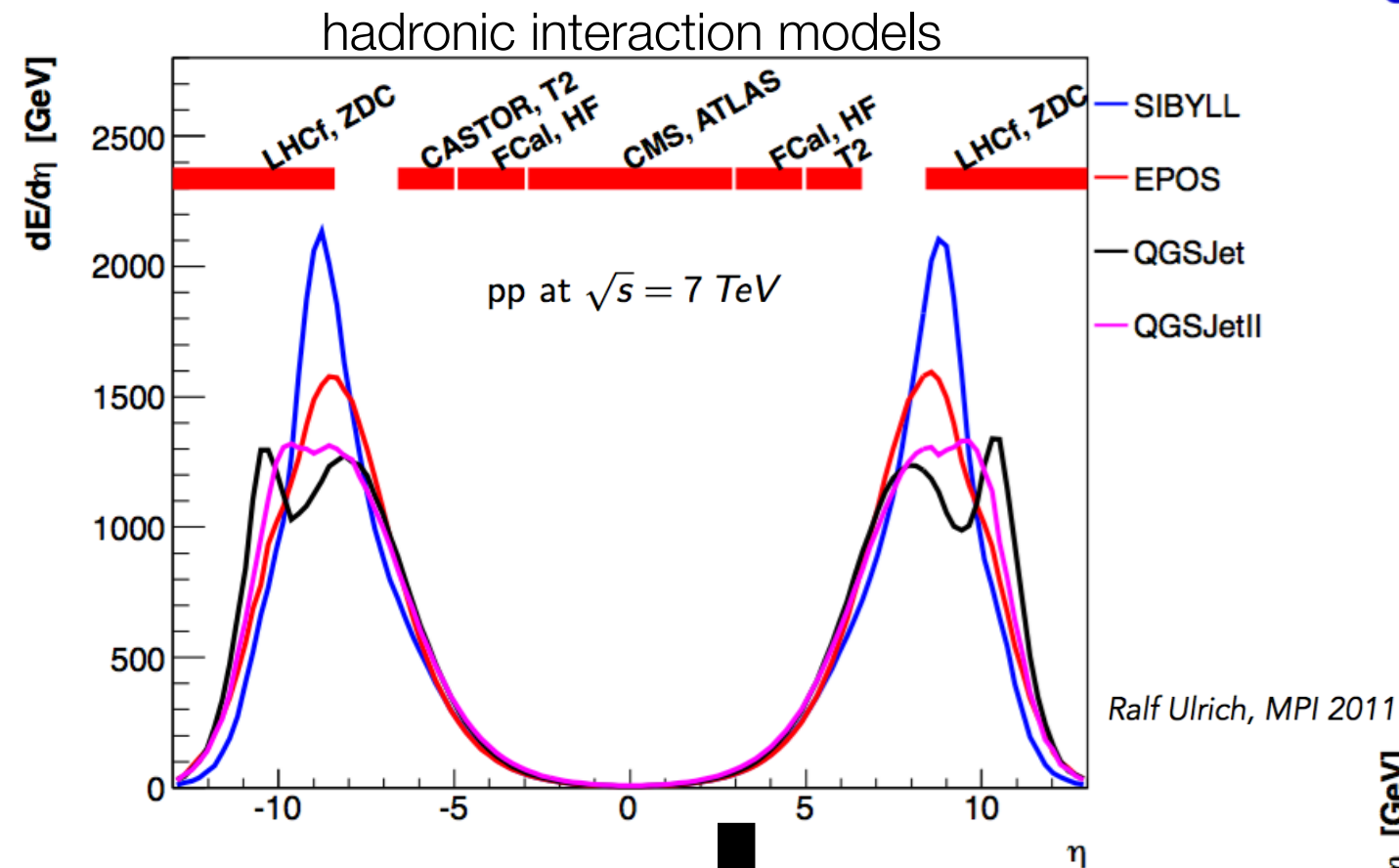
SOPHIA (Mücke, RE, Rachen, Protheroe, Stanev)

Ralph Engel

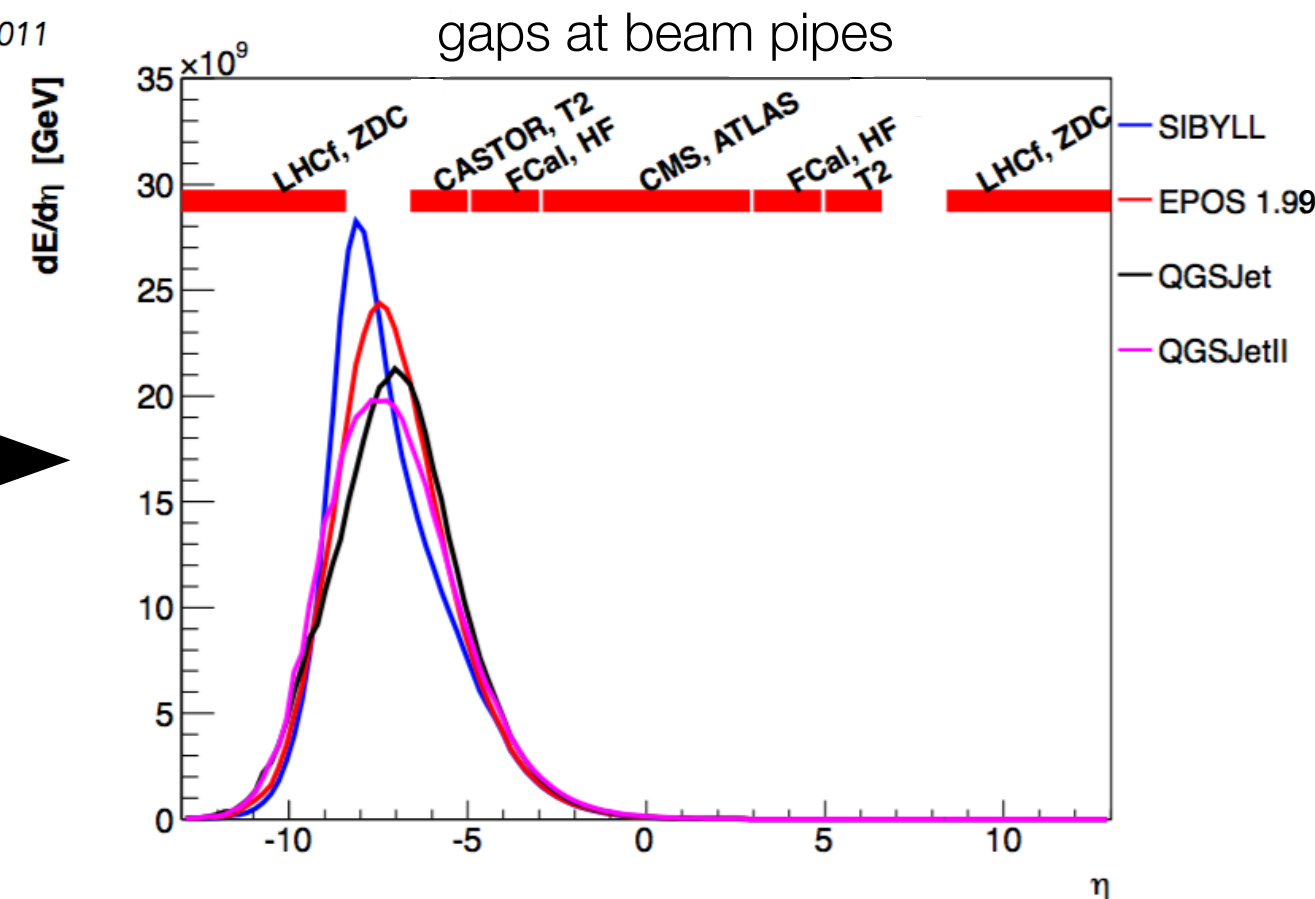
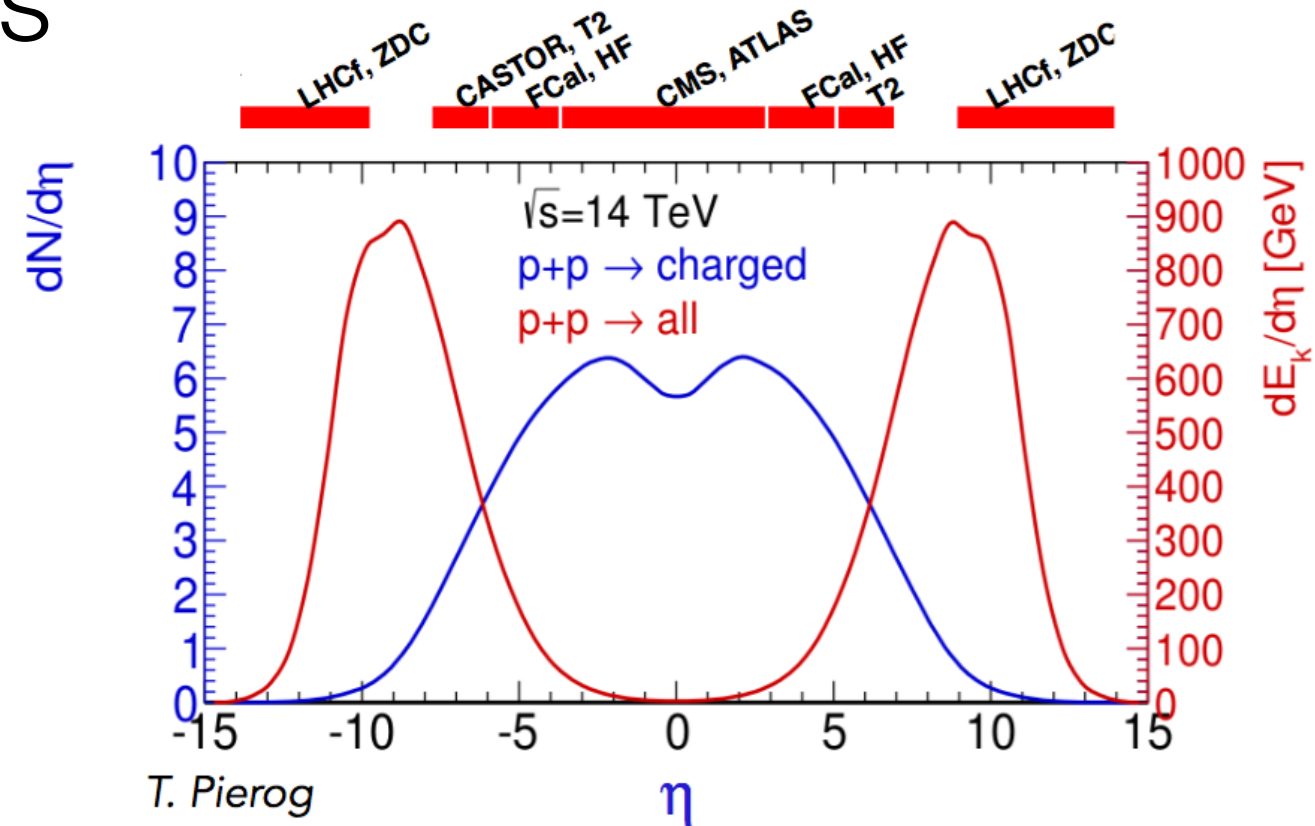


# hadronic interaction models

## treating the forward region



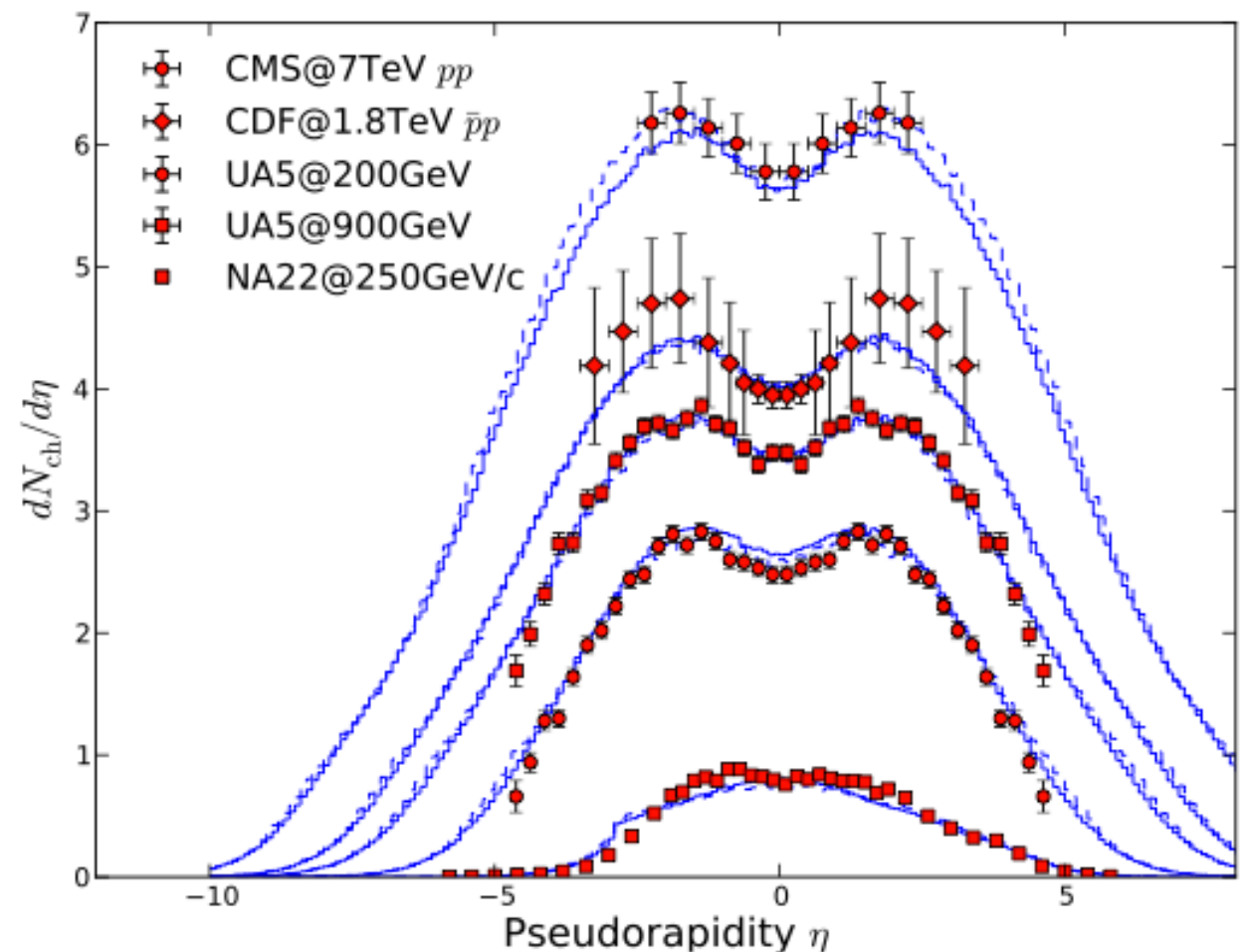
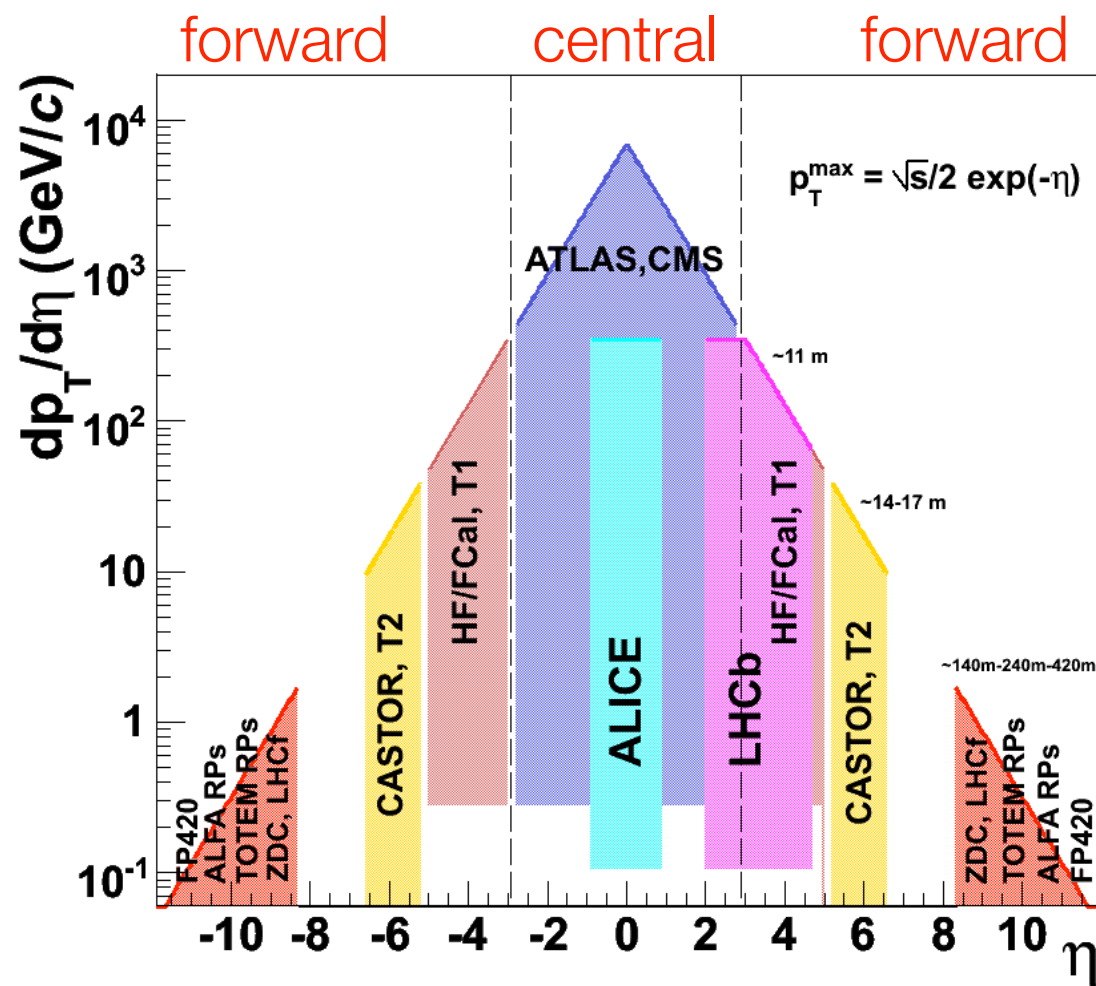
boosted from CM to air  
shower reference frame



# hadronic interaction models

## treating the forward region

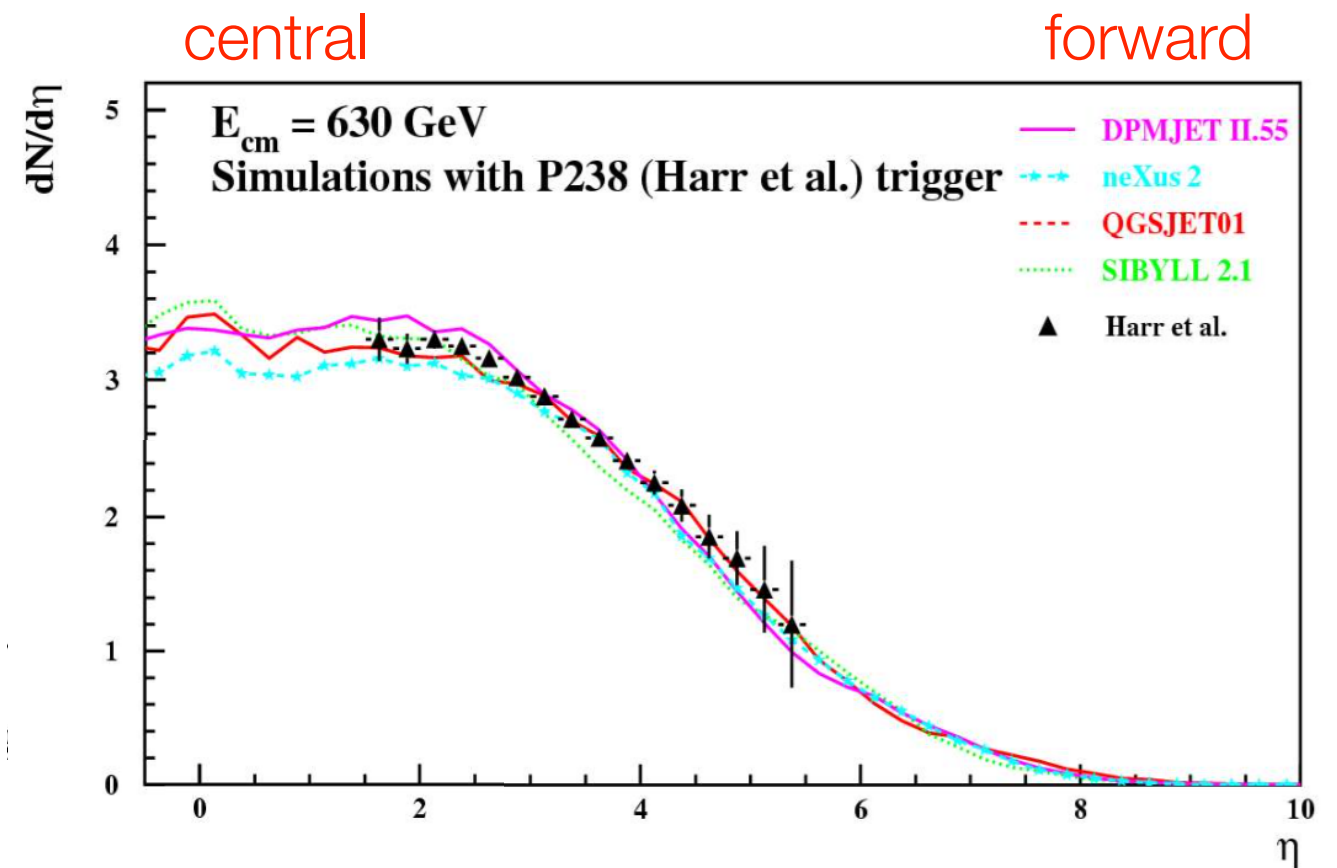
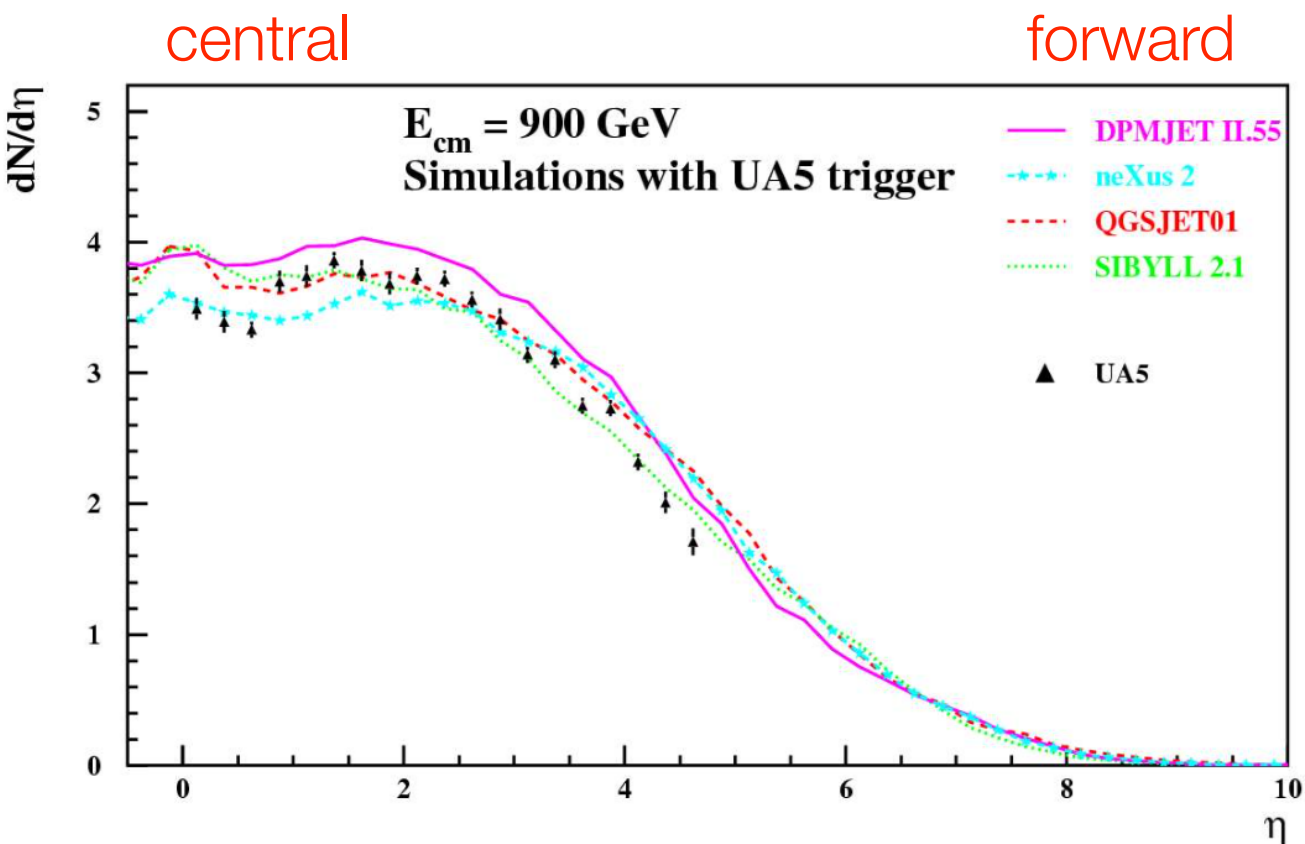
- ▶ **forward region** the most relevant in cosmic rays
- ▶ models **tuned** to accelerator measurements and extrapolated



# hadronic interaction models

## tuning to accelerator data

proton-antiproton @CERN-SPS



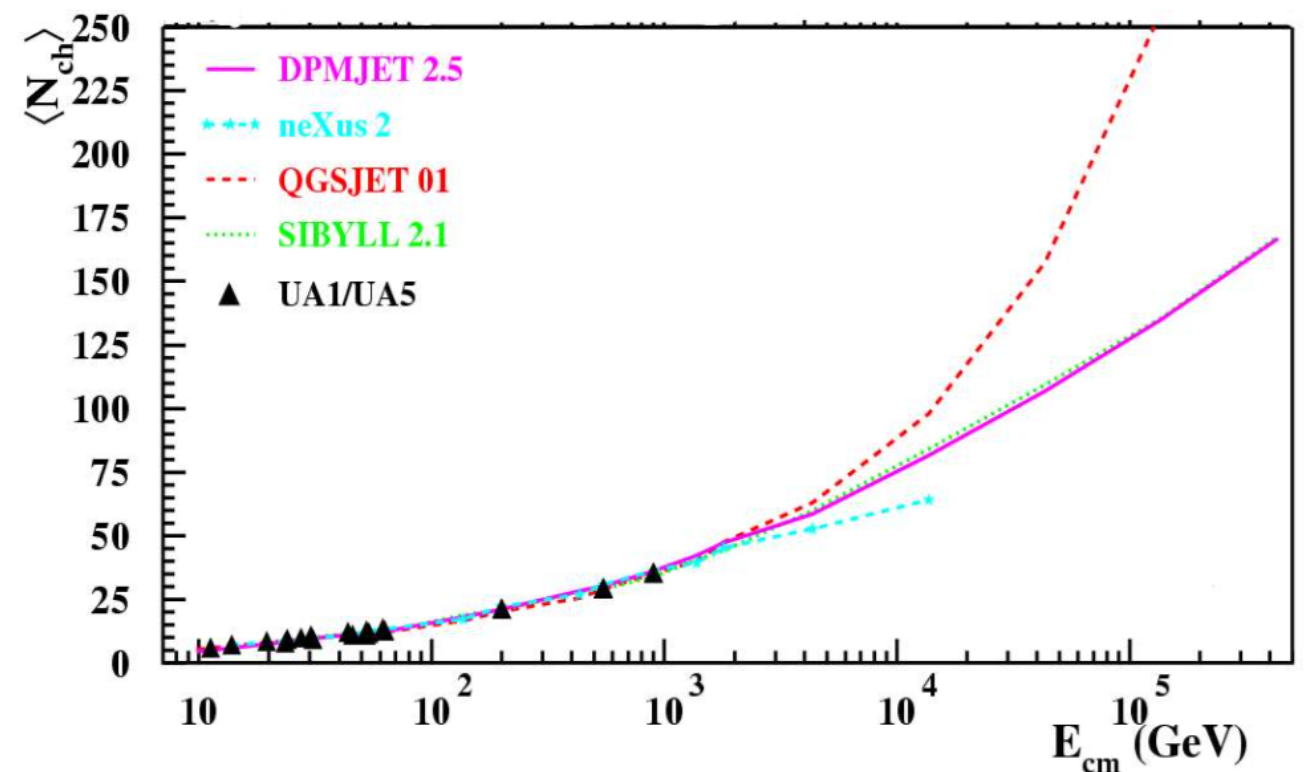
pseudo-rapidity distribution of charged particles

# hadronic interaction models

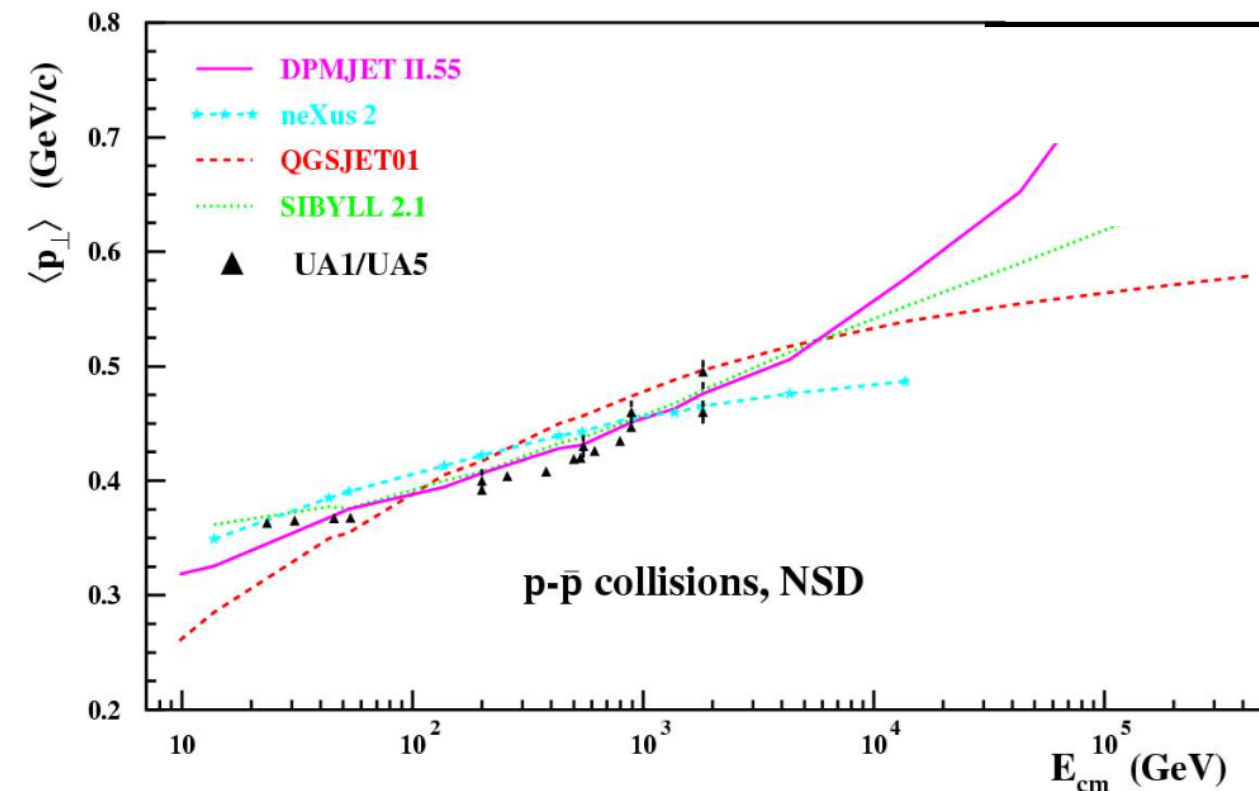
## tuning to accelerator data

proton-antiproton @CERN-SPS & Tevatron

mean charged particle multiplicity



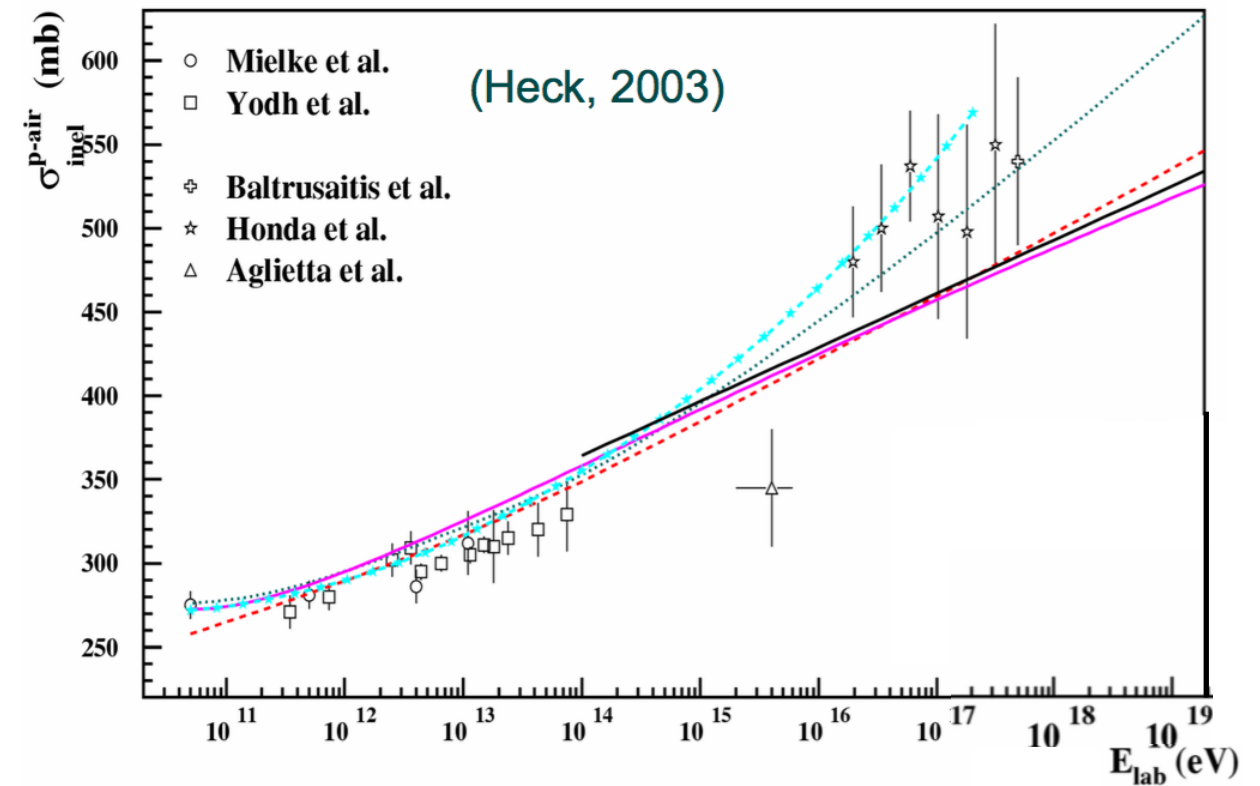
mean transverse momentum



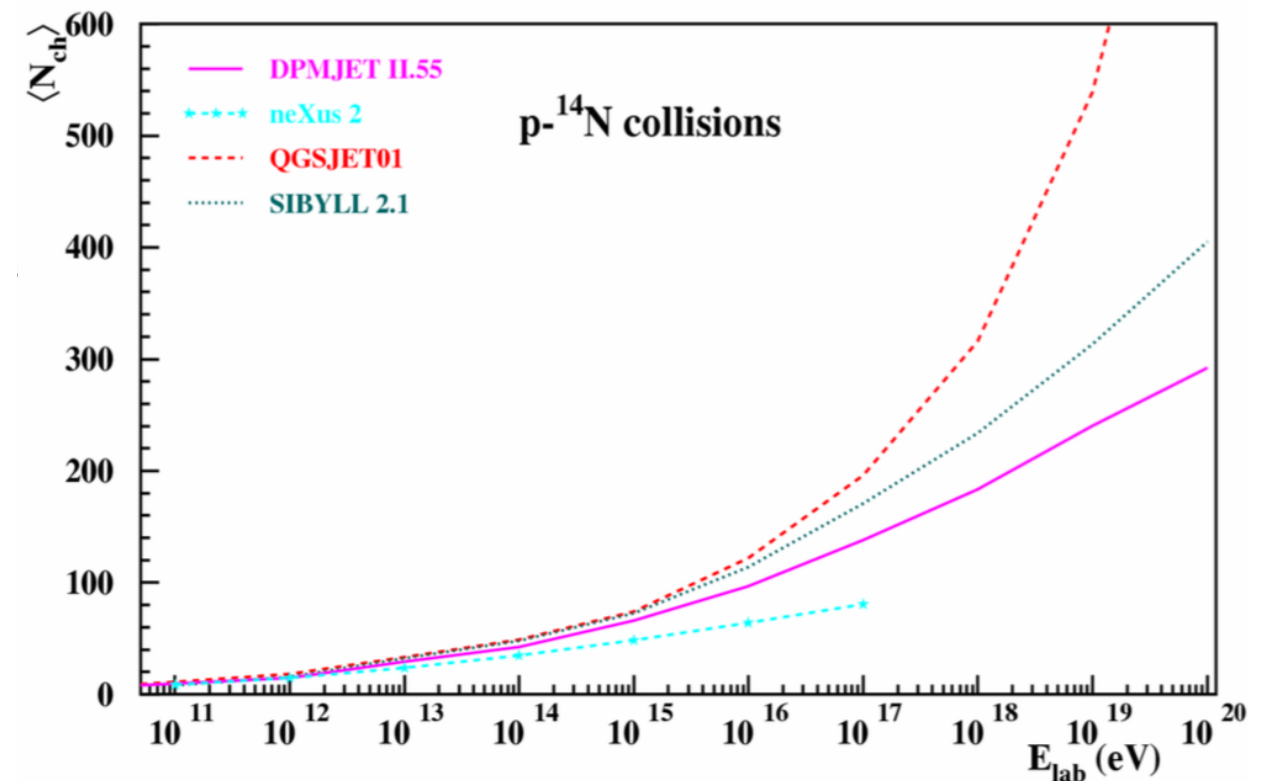
# hadronic interaction models

## tuning to accelerator data

p-air production cross section  
& model predictions



secondary charged particle  
multiplicity predictions

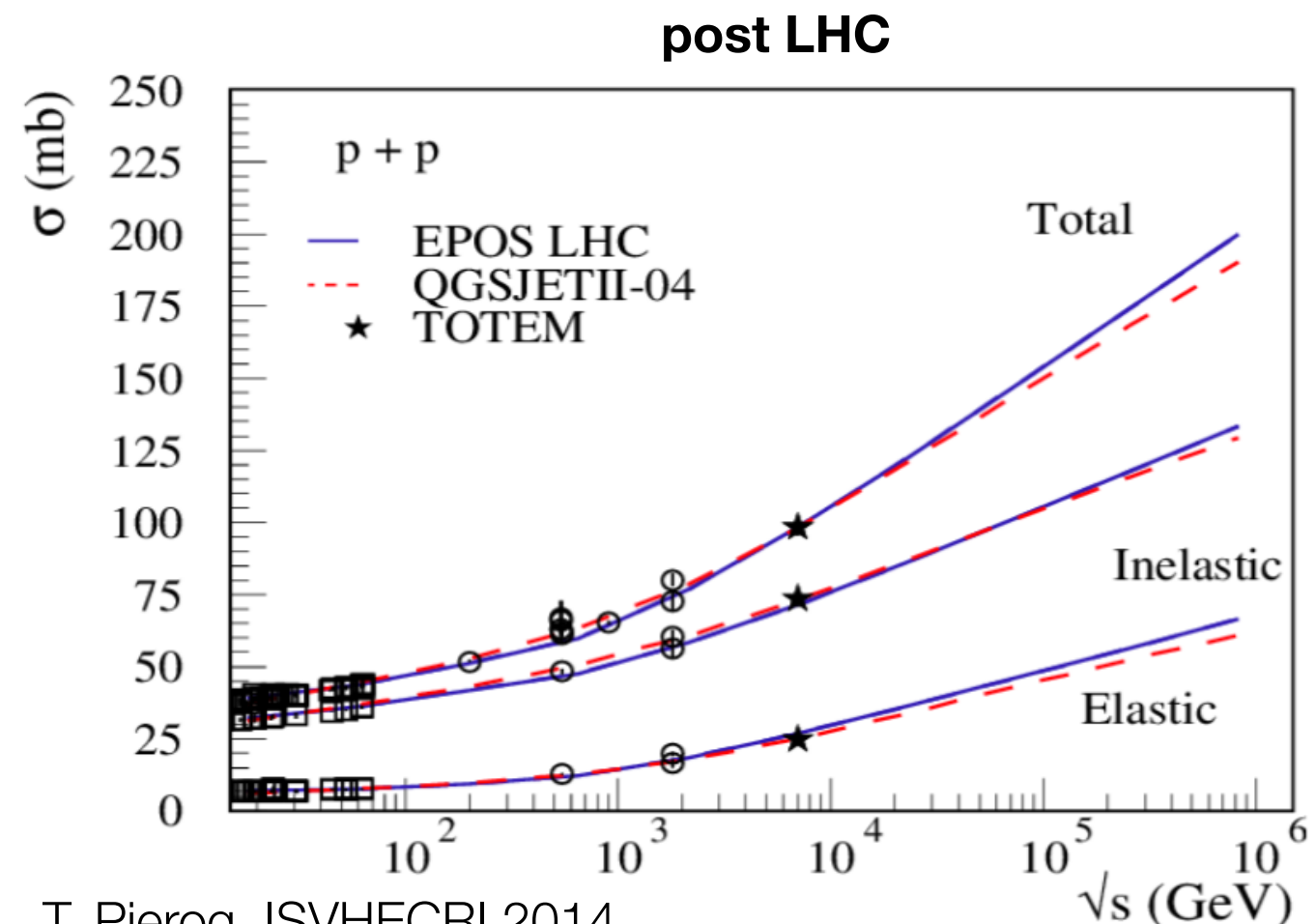
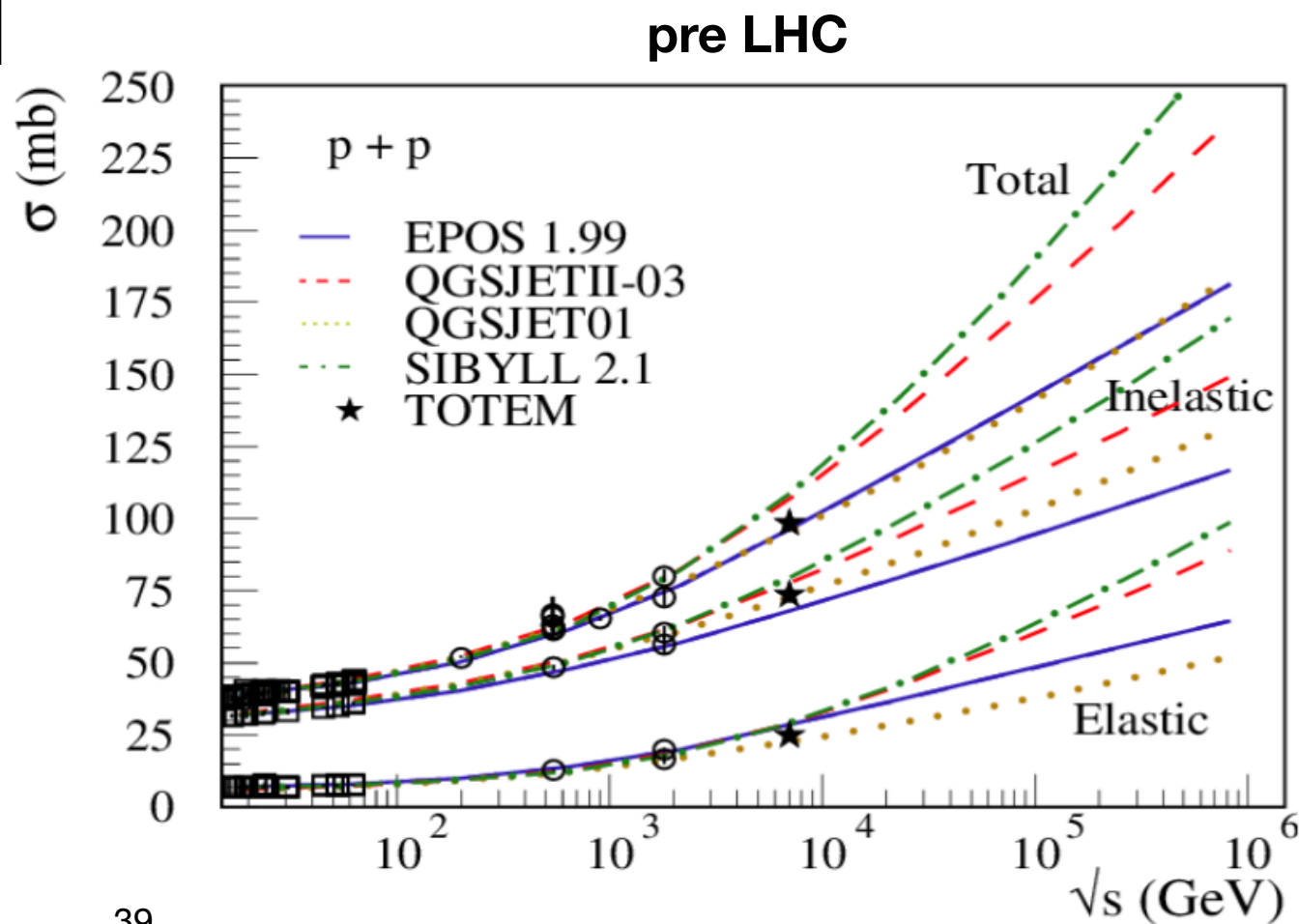




# hadronic interaction models

## treating the forward region

- ▶ **LHC** is providing good high energy data in wide range of pseudo-rapidity
- ▶ ... although with important gaps ... still



# primary cosmic rays

## interactions in the atmosphere

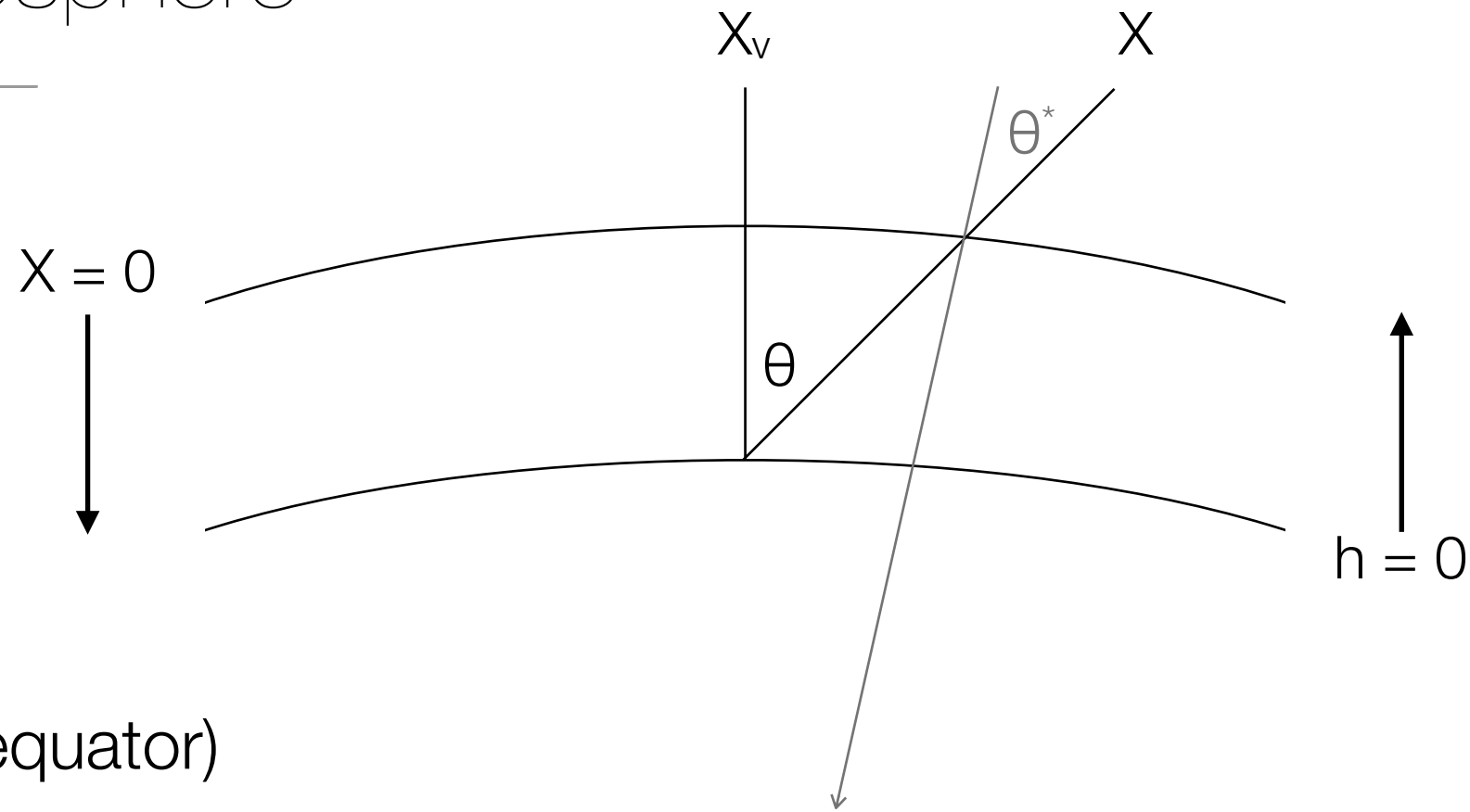
$$X_v = \int_h^\infty \rho_{air}(h') dh'$$

$$X \approx X_v \cos \theta^*$$

- isothermal atmosphere  
(7-50 km @poles - 17-50 km @equator)

$$\rho(h) \simeq \rho_0 e^{-h/h_0} \quad \rho_0 \simeq 2.03 \times 10^{-3} \text{ g cm}^{-3} \quad h_0 \simeq 6.4 \text{ km}$$

$$X(h) \simeq \rho_0 h_0 e^{-h/h_0} \simeq h_0 \rho(h)$$

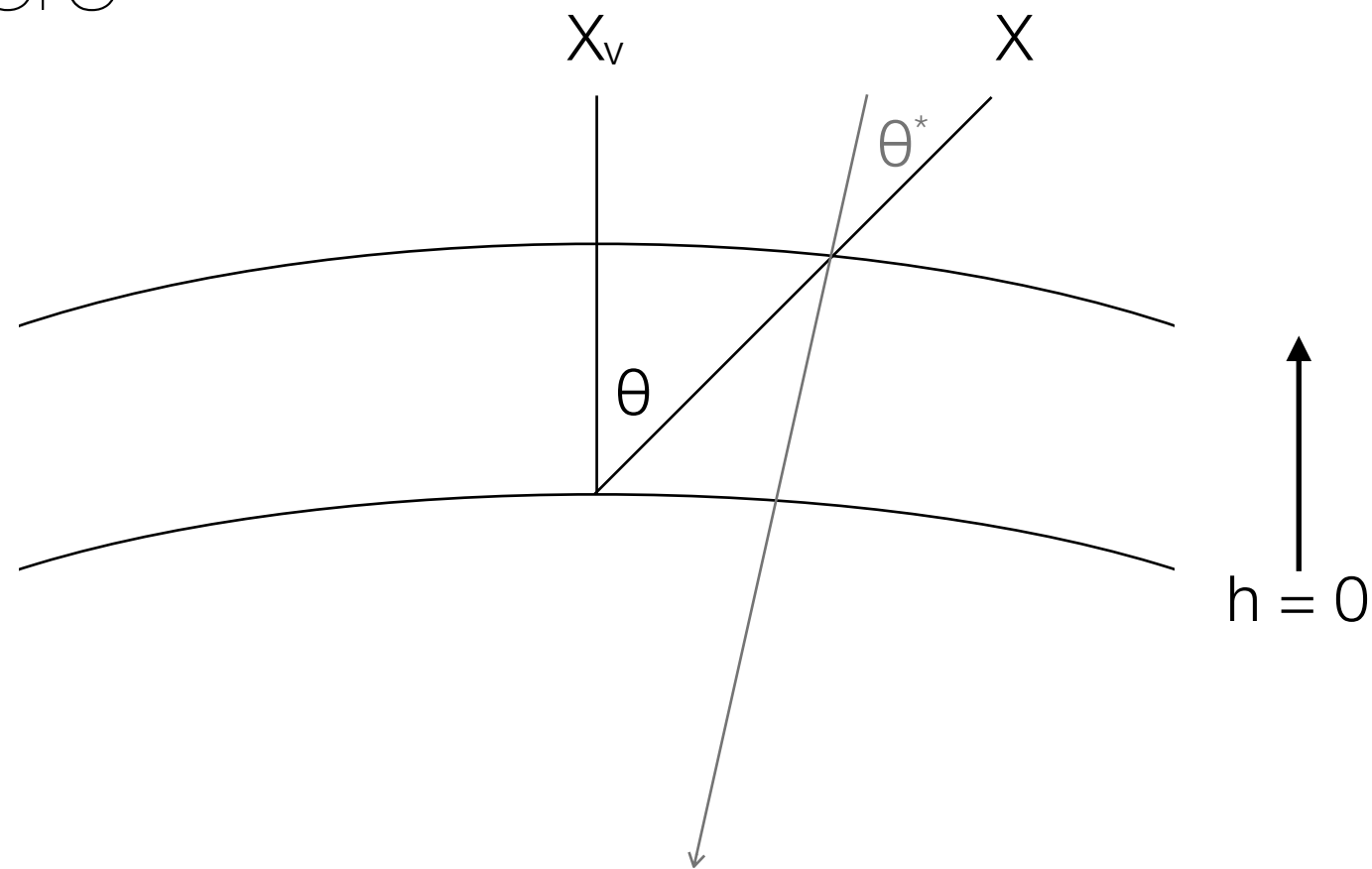


# primary cosmic rays

## interactions in the atmosphere

$$X(h) \simeq \rho_0 h_0 e^{-h/h_0} \simeq h_0 \rho(h)$$

$X = 0$   
↓



- hadronic interactions

$$\lambda_N = \frac{\rho_{air}}{n_N \sigma_N^{air}}$$

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$$

$$\mu^\pm \rightarrow e^\pm + \nu_e(\bar{\nu}_e) + (\bar{\nu}_\mu)\nu_\mu$$

$$K^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu) \quad (64 \%)$$

$$K^\pm \rightarrow \pi^\pm + \pi^0 \quad (21 \%)$$

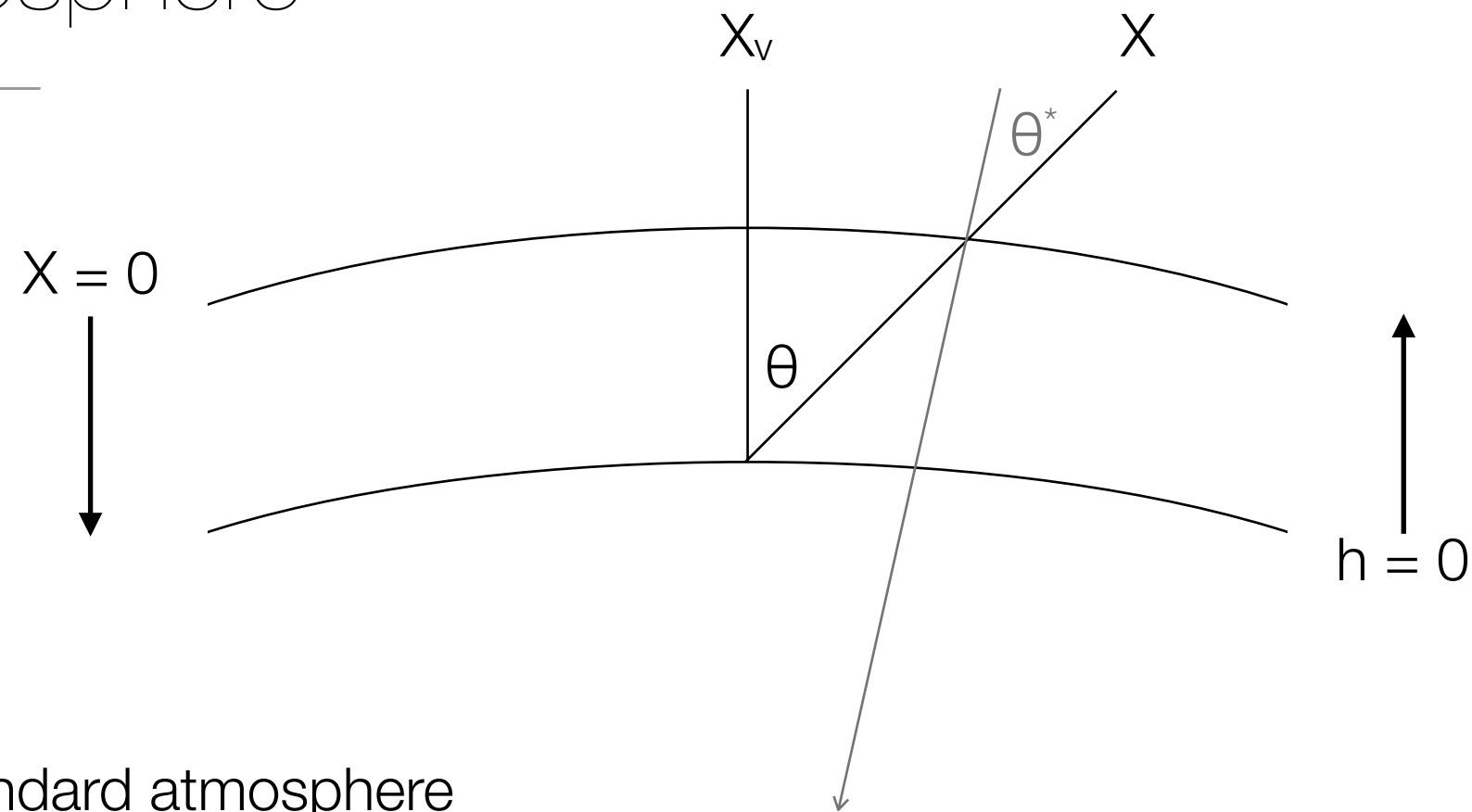
# primary cosmic rays

## interactions in the atmosphere

$$\lambda_N = \frac{\rho_{air}}{n_N \sigma_N^{air}}$$

$$\begin{aligned}\lambda_{int,p} &\approx 90 \text{ g cm}^{-2} \\ \lambda_{int,Fe} &\approx 5 \text{ g cm}^{-2} \\ \lambda_{int,\pi} &\approx 120 \text{ g cm}^{-2} \\ \lambda_{int,K} &\approx 12 \text{ g cm}^{-2}\end{aligned}$$

$$X_v = \int_h^\infty \rho_{air}(h') dh'$$



US standard atmosphere

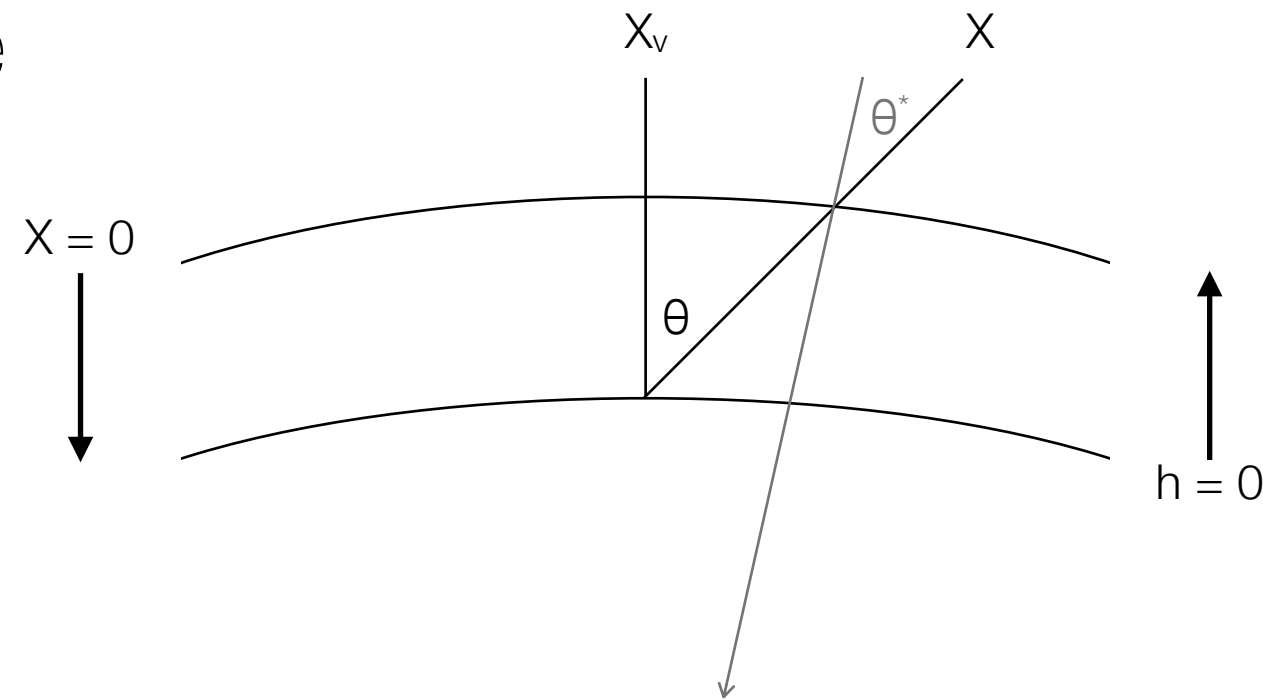
Altitude (km)	Vertical depth (g/cm <sup>2</sup> )	Local density (10 <sup>-3</sup> g/cm <sup>3</sup> )	Molière unit (m)	Electron Cherenkov threshold (MeV)	Cherenkov angle (°)
40	3	$3.8 \times 10^{-3}$	$2.4 \times 10^4$	386	0.076
30	11.8	$1.8 \times 10^{-2}$	$5.1 \times 10^3$	176	0.17
20	55.8	$8.8 \times 10^{-2}$	$1.0 \times 10^3$	80	0.36
15	123	0.19	478	54	0.54
10	269	0.42	223	37	0.79
5	550	0.74	126	28	1.05
3	715	0.91	102	25	1.17
1.5	862	1.06	88	23	1.26
0.5	974	1.17	79	22	1.33
0	1,032	1.23	76	21	1.36

# primary cosmic rays

## interactions in the atmosphere

- hadronic interactions

$$\lambda_i = \frac{\rho_{air}}{\rho_i \sigma_i^{air}}$$



- meson decay

$$\frac{1}{d_i} = \frac{\epsilon_i}{E_i X \cos \theta^*}$$

$$\epsilon_i = \frac{m_i c^2}{c \tau_i} h_0(T)$$

- competition between interaction & decay**  $\frac{1}{d_i} \approx \frac{1}{\lambda_i}$  which depends on  $\frac{\epsilon_i}{E_i}$

$$\epsilon_{\pi^0} = 3.5 \times 10^{10} GeV$$

$$\epsilon_{K_L^0} = 210 GeV$$

$$\epsilon_{D^0} = 9.2 \times 10^7 GeV$$

$$\epsilon_{\pi^\pm} = 115 GeV$$

$$\epsilon_{K^\pm} = 850 GeV$$

$$\epsilon_{D^\pm} = 4.3 \times 10^7 GeV$$

strange

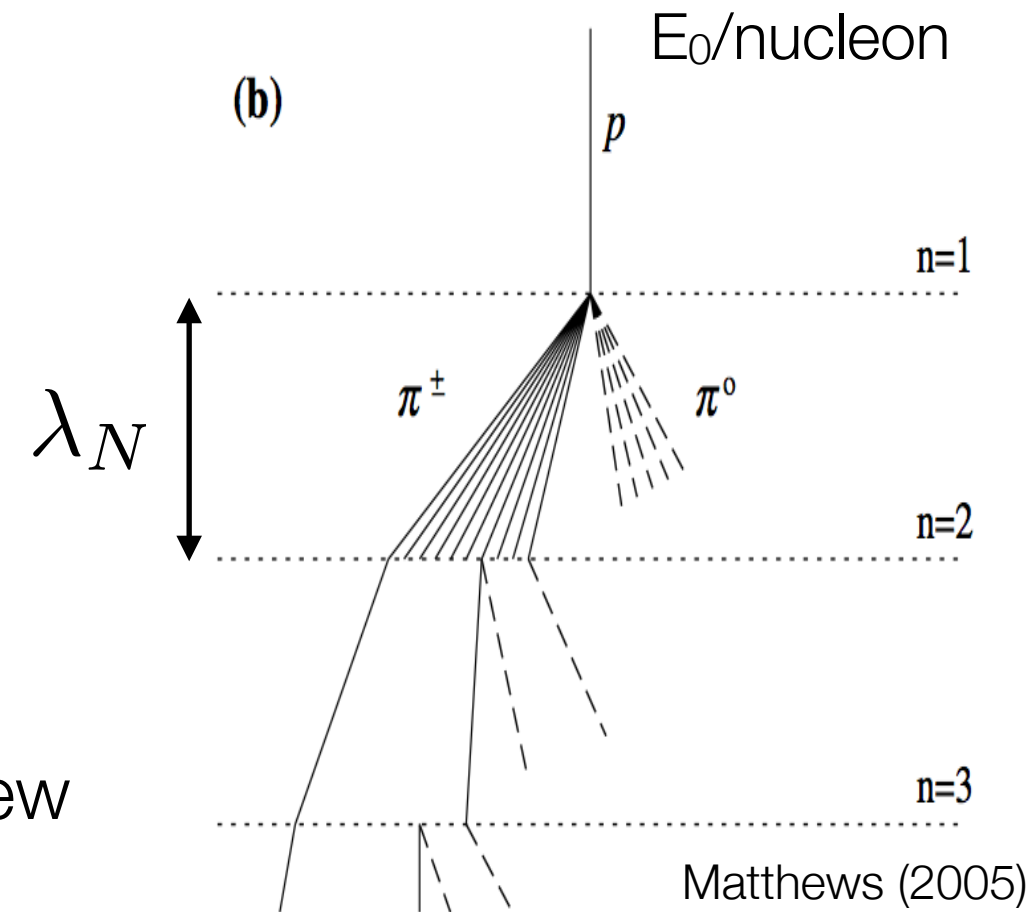
charm



# hadronic showers

## Heitler model

- nucleons ( $E_0$ ) ignites showers producing  $\pi$ 's
  - ▶  $\pi^0 \rightarrow \gamma\gamma$  (fast electromagnetic process)
  - ▶  $\pi^\pm \rightarrow \mu^\pm + \nu_\mu$  (slow weak process) OR initiate new cascades until  $E_c$



$$\lambda_N = \frac{\rho_{air}}{n_N \sigma_N^{air}}$$

- assume one muon produced for each hadron

$$N_\mu = \left( \frac{E_0}{E_c^\pi} \right)^\alpha \approx 10^4 \left( \frac{E_0}{1 \text{ PeV}} \right)^\alpha$$

$$\alpha = \frac{\ln(n_{ch})}{\ln(n_{tot})} \approx 0.85 - 0.92$$

# hadronic showers

## superposition model

- atomic nucleus of mass  $A$  and energy  $E_0 \approx A$   
nucleons of energy  $E_0/A$

$$N_{\mu}^A = A \left( \frac{E_0}{AE_c^{\pi}} \right)^{\alpha} = A^{1-\alpha} N_{\mu}$$

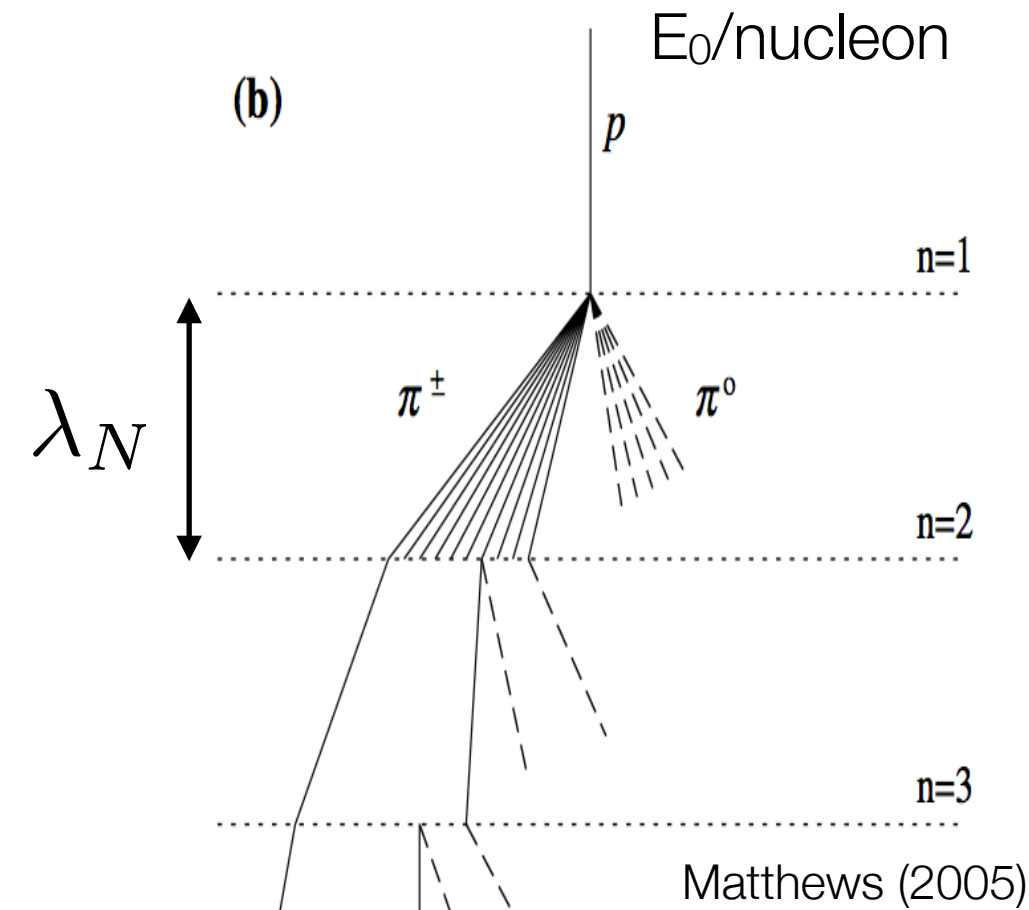
$$N_{\mu}(> E_{\mu}) \approx A \times \frac{14.5 \text{ GeV}}{E_{\mu} \cos \theta} \left( \frac{E_0}{AE_{\mu}} \right)^{0.757} \left( 1 - \frac{AE_{\mu}}{E_0} \right)^{5.25} \quad \text{Elbert Formula}$$

- mass-dependent quantities

$$X_{max}^A \approx \lambda_{eff} \ln(E_0/A)$$

$$E_0 \approx 0.85(N_e + 25N_{\mu})$$

$\lambda_{eff}$  the effective interaction length for pions

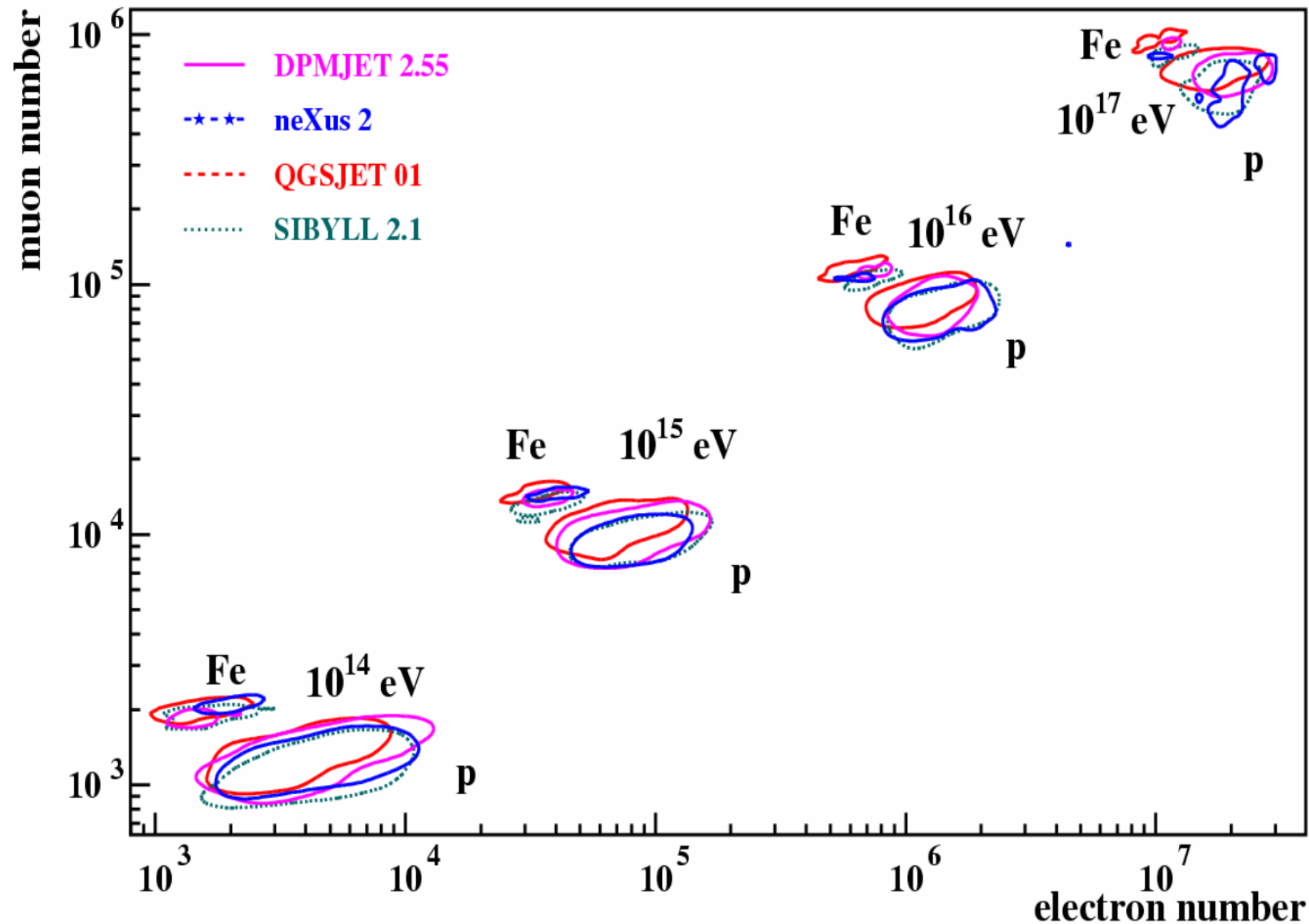


$$\lambda_N = \frac{\rho_{air}}{n_N \sigma_N^{air}}$$

# hadronic showers

## energy & mass composition

Ralf Engel

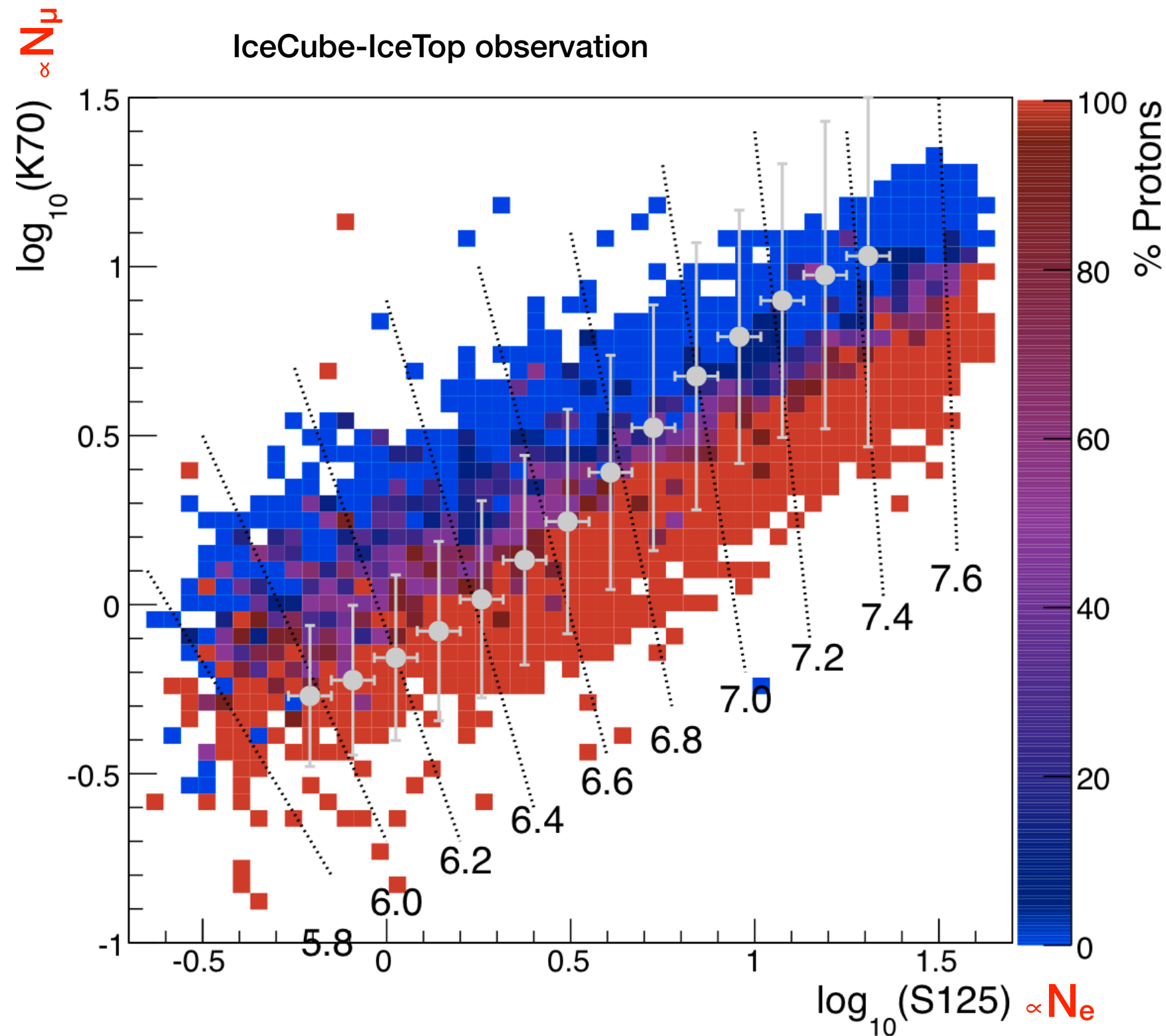


$$N_{\mu}^A = A^{1-\alpha} \left( \frac{E_0}{E_c^{\pi}} \right)^{\alpha}$$

$$N_e^{\gamma} \propto \frac{0.31}{\sqrt{\ln(E_0/\epsilon_c)}} e^{-X/\lambda_{rad}}$$

# hadronic showers

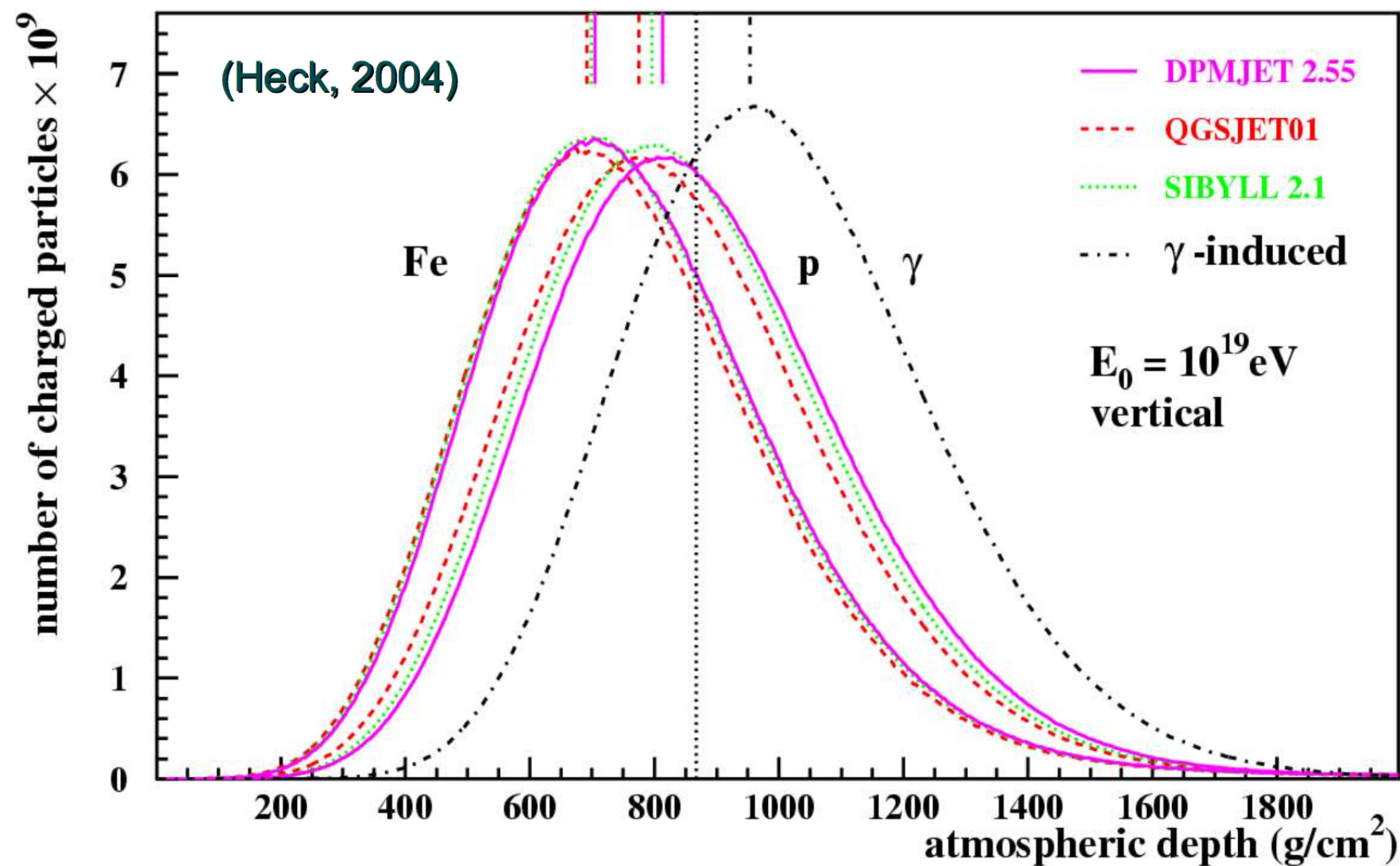
energy & mass composition



$$N_\mu^A = A^{1-\alpha} \left( \frac{E_0}{E_c^\pi} \right)^\alpha$$

$$N_e^\gamma \propto \frac{0.31}{\sqrt{\ln(E_0/\epsilon_c)}} e^{-X/\lambda_{rad}}$$

# hadronic vs. electromagnetic showers

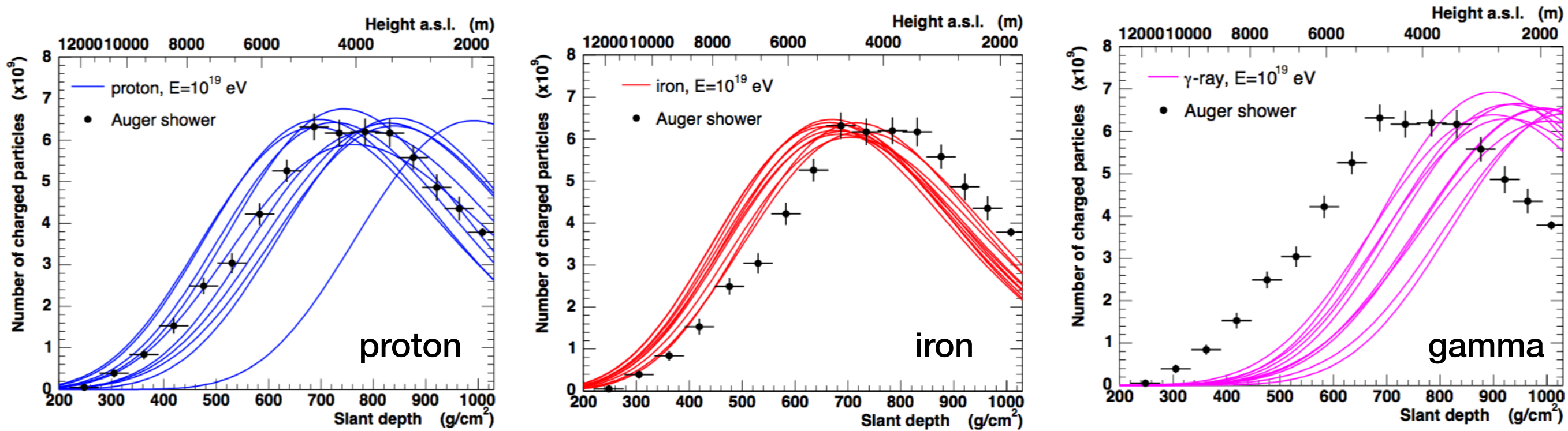


$$X_{max}^{EM} \approx \lambda_{int} \ln(E_0/\epsilon_c)$$

$$X_{max}^A \approx \lambda_{eff} \ln(E_0/A)$$



# hadronic vs. electromagnetic showers



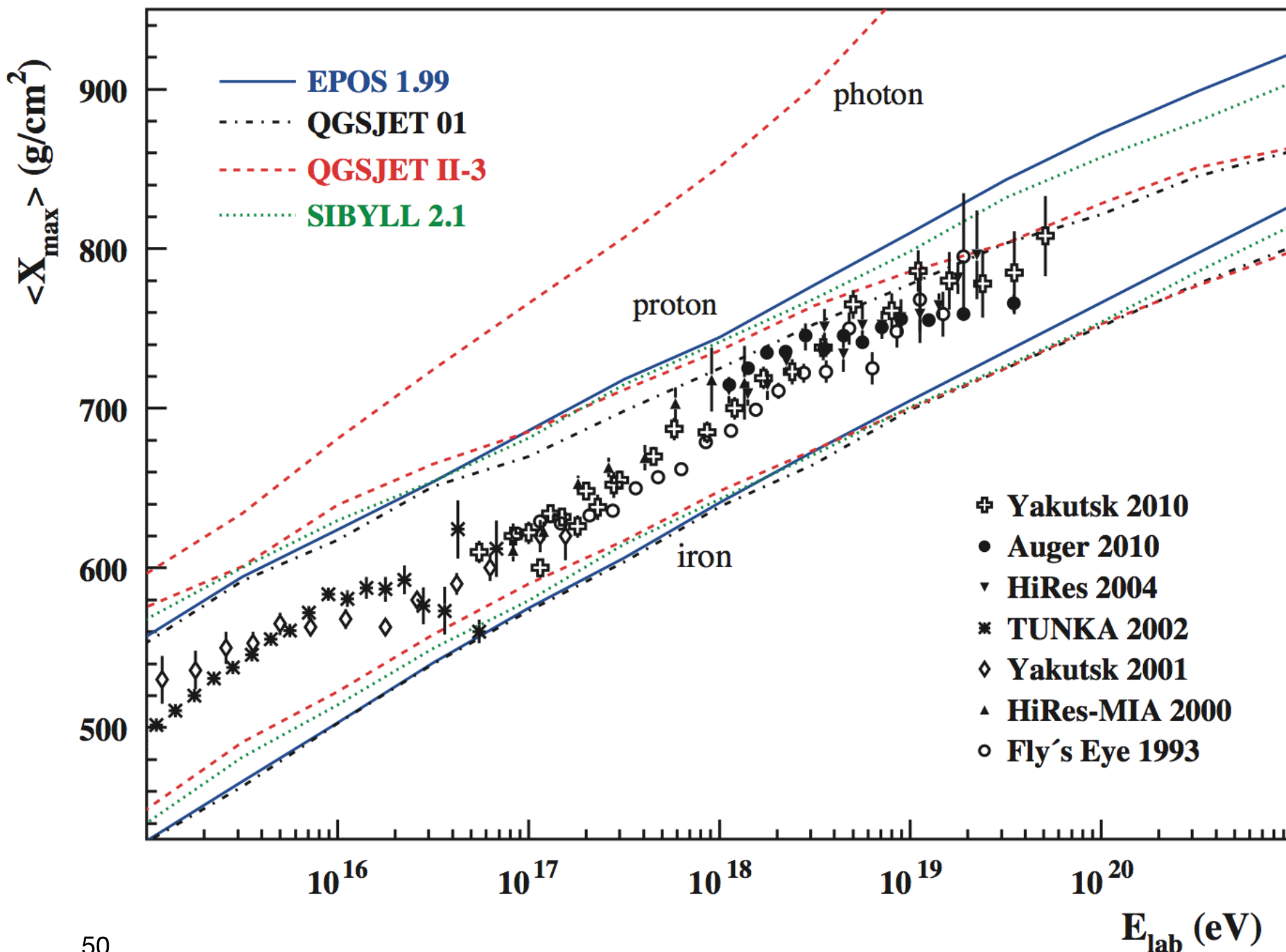
$$X_{max}^{EM} \approx \lambda_{int} \ln(E_0/\epsilon_c)$$

$$X_{max}^A \approx \lambda_{eff} \ln(E_0/A)$$

# hadronic showers

## energy & mass composition

Engel, Pierog & Heck, ARNPS 2011



$$X_{max}^{EM} \approx \lambda_{int} \ln(E_0/\epsilon_c)$$

$$X_{max}^A \approx \lambda_{eff} \ln(E_0/A)$$

# elongation rate theorem

constraint on  
elongation rate of  
hadron-induced showers

- elongation rate (change of  $X_{\max}$  vs. energy)

$$D_e = \frac{\langle dX_{\max} \rangle}{d \ln E}$$

$$D_{10} = \frac{\langle dX_{\max} \rangle}{d \log E} = \ln(10) D_e$$

- photon-induced shower

$$\langle X_{\max}^{\gamma} \rangle \approx \lambda_{int} \ln(E_0/\epsilon_c)$$

$$D_{10}^{\gamma} = \ln(10) \lambda_{int} \approx 84 \text{ g/cm}^2$$

- hadron-induced shower (**elongation theorem**)

$$D_{10}^{had} = D_{10}^{\gamma} (1 - B_n - B_{\lambda})$$

$$B_n = \frac{d \ln \langle n(E) \rangle}{d \ln E}$$

Lindsey & Watson PRL 46, 459, 1981

$$B_{\lambda} = -\frac{\lambda_{int}}{\lambda_{rad}} \frac{d \ln \lambda_{int}}{d \ln E}$$

# hadronic showers

## cascade equations

---

- particle flux  $\Phi(E, X) = \frac{dN}{dt dA dE}$
- evolution with column depth (for protons)

$$\frac{\partial \Phi_p(E, X)}{\partial X} = - \frac{\Phi_p(E, X)}{\lambda_{int,p}(E)} \quad \text{loss}$$

$$+ \int_E^\infty \frac{1}{\lambda_{int,p}(E)} \Phi_p(\tilde{E}, X) \frac{dn_{p \rightarrow p}}{dE}(\tilde{E}, E) d\tilde{E} \quad \text{production}$$


---

- reminds you anything familiar ?

$$\frac{\partial n_\gamma(E, t)}{\partial t} = -\sigma_0 n_\gamma(E, t) + \int_0^1 \frac{dv}{v} \varphi(v) n_e\left(\frac{E}{v}, t\right)$$

$$t = \frac{X}{\lambda_{rad}} \quad v = \frac{E_e}{E_\gamma}$$

$$n_\gamma(E, t) = K r_\gamma^{(1)}(s) E^{-(s+1)} e^{\lambda_1(s) t} \quad \text{ansatz}$$

# hadronic showers

## cascade equations

---

- evolution with column depth (for protons)

$$\frac{\partial \Phi_p(E, X)}{\partial X} = - \frac{\Phi_p(E, X)}{\lambda_{int,p}(E)} \quad \text{loss}$$

$$+ \int_E^\infty \frac{1}{\lambda_{int,p}(E)} \Phi_p(\tilde{E}, X) \frac{dn_{p \rightarrow p}}{dE}(\tilde{E}, E) d\tilde{E} \quad \text{production}$$

- ansatz  $\Phi(E, X) = A(X) E^{-(\gamma+1)}$

- factorized equation

$$E^{-(\gamma+1)} \frac{\partial A_p(X)}{\partial X} = -E^{-(\gamma+1)} \frac{A_p(X)}{\lambda_{int,p}} \left[ 1 - \int_E^\infty \left( \frac{\tilde{E}}{E} \right)^{-(\gamma+1)} \frac{dn_{p \rightarrow p}}{dE} d\tilde{E} \right]$$



# hadronic showers

## cascade equations

---

- factorized equation

$$E^{-(\gamma+1)} \frac{\partial A_p(X)}{\partial X} = -E^{-(\gamma+1)} \frac{A_p(X)}{\lambda_{int,p}} \left[ 1 - \int_E^\infty \left( \frac{\tilde{E}}{E} \right)^{-(\gamma+1)} \frac{dn_{p \rightarrow p}}{dE} d\tilde{E} \right]$$

- assume scaling

$$\frac{dn_{p \rightarrow p}}{dE}(E, \tilde{E}) = \frac{1}{\tilde{E}} \frac{dn_{p \rightarrow p}}{dx_{LAB}} \quad dx_{LAB} = \frac{E}{\tilde{E}}$$

$$\frac{\partial A_p(X)}{\partial X} = -\frac{A_p(X)}{\lambda_{int,p}} \left[ 1 - \int_0^1 x_{LAB}^\gamma \frac{dn_{p \rightarrow p}}{dx_{LAB}} dx_{LAB} \right]$$

$$\frac{\partial A_p(X)}{\partial X} = -\frac{A_p(X)}{\lambda_{int,p}} [1 - Z_{pp}]$$

$Z_{pp}$  - spectrum-weighted moment

# hadronic showers

## cascade equations

---

- equation

$$\frac{\partial A_p(X)}{\partial X} = -\frac{A_p(X)}{\lambda_{int,p}} [1 - Z_{pp}]$$

- solution (inclusive flux can be numerically calculated)

$$\Phi_p(E, X) = A_0 e^{-X/\Lambda_{pp}} E^{-(\gamma+1)} \quad \Lambda_{pp} = \frac{\lambda_{int,p}}{1 - Z_{pp}} \quad \begin{array}{l} \text{attenuation} \\ \text{length} \end{array}$$

- in reality the hadronic cascade equations contains **many loss and coupled production channels** and the Z-factors need to be determined using **collider data** or **calculated multiplicity spectra**

# hadronic showers

## cascade equations

---

Anatoli Fedynitch, ISVHECRI 2014

MCEq cascade equations integrator

<https://github.com/afedynitch/MCEq>

- in reality the hadronic cascade equations contains **many loss and coupled production channels** and the Z-factors need to be determined using **collider data** or **calculated multiplicity spectra**

LOSSES

$$\frac{d\Phi_h(E, X)}{dX} =$$

$$\begin{aligned} & - \frac{\Phi_h(E, X)}{\lambda_{int,h}^h(E)} \\ & - \frac{\Phi_h(E, X)}{\lambda_{dec,h}(E, X)} \end{aligned}$$

loss due to interaction

loss due to decays

$$+ \frac{\partial}{\partial E} (b(E) \Phi_h(E, X))$$

continuous energy loss

GAINS

$$\begin{aligned} & + \sum_l S(l \rightarrow h, E) \\ & + \sum_l S^D(h \rightarrow h, E) \end{aligned}$$

production due to interactions of other particles

production due to decays of other particles

# hadronic showers

## couplings in cascade equations

- assuming p, n and pion system only
- pion inclusive flux

$$\frac{d\Phi_\pi}{dX} = \underbrace{\left( -\frac{\Phi_\pi}{\lambda_{int,p}} - \frac{\Phi_\pi}{\lambda_{dec,\pi}} \right)}_{\substack{\text{loss} \\ \text{feeds} \\ \text{cascade}}} + \underbrace{\left( Z_{\pi\pi} \frac{\Phi_\pi}{\lambda_{int,\pi}} + \underbrace{\left( Z_{p\pi} \frac{\Phi_p}{\lambda_{int,p}} + Z_{n\pi} \frac{\Phi_n}{\lambda_{int,n}} \right)}_{\substack{\text{nucleon-pion coupling}}} \right)}_{\substack{\text{production}}}$$

- muon inclusive flux  $\frac{d\Phi_\mu}{dX} = -\frac{\Phi_\mu}{\lambda_{dec,\mu}} + Z_{\pi \rightarrow \mu}^{\text{decay}} \frac{\Phi_\pi}{\lambda_{dec,\pi}}$
- neutrino inclusive flux  $\frac{d\Phi_\nu}{dX} = Z_{\pi \rightarrow \nu}^{\text{decay}} \frac{\Phi_\pi}{\lambda_{dec,\pi}}$
- competition between decay and interaction

# hadronic showers

## inclusive neutrino flux

---

Anatoli Fedynitch

- low energy (no pion interaction term)

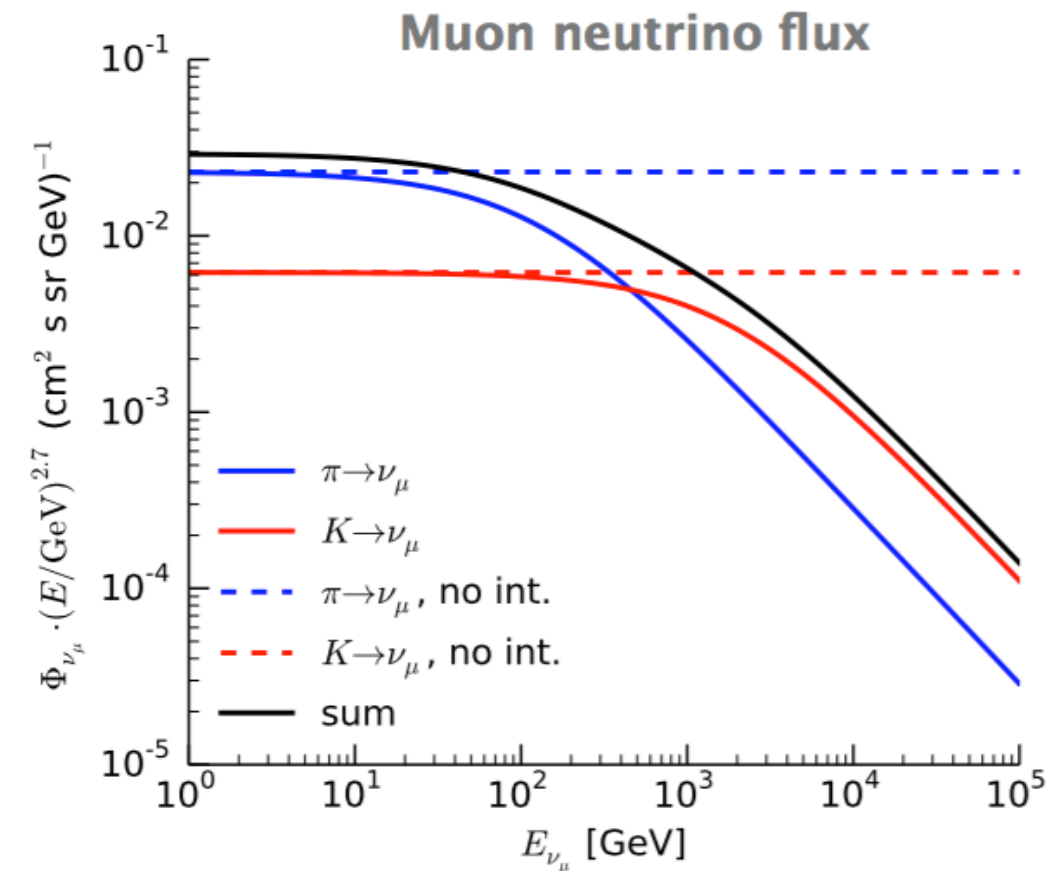
$$\Phi_\nu(E) \propto \Phi_\pi \propto E^{-\gamma_{CR}}$$

- high energy (no pion decay term)

$$\Phi_\nu(E) \propto \Phi_\pi \propto E^{-(\gamma_{CR}+1)}$$

- interpolation

$$\frac{\Phi_{LE} \Phi_{HE}}{\Phi_{LE} + \Phi_{HE}}$$





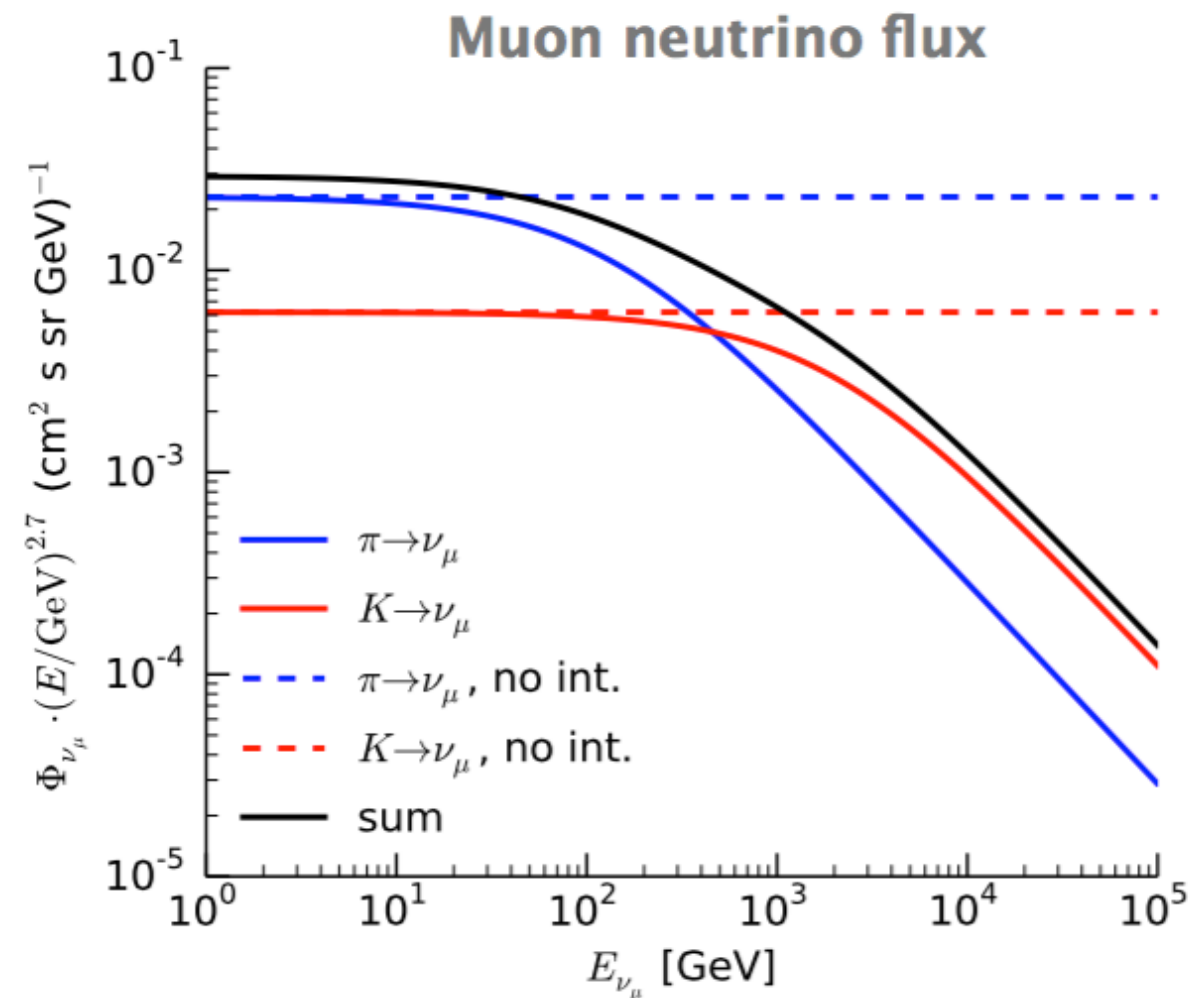
# hadronic showers

## inclusive neutrino flux

---

- neutrino flux

$$\Phi_\nu(E) = \frac{\Phi_N(E)}{1 - Z_{NN}} \left( \frac{A_{\pi\nu}}{1 + B_{\pi\nu} E \cos\theta^* / \epsilon_\pi} + \frac{A_{K\nu}}{1 + B_{K\nu} E \cos\theta^* / \epsilon_K} \right)$$

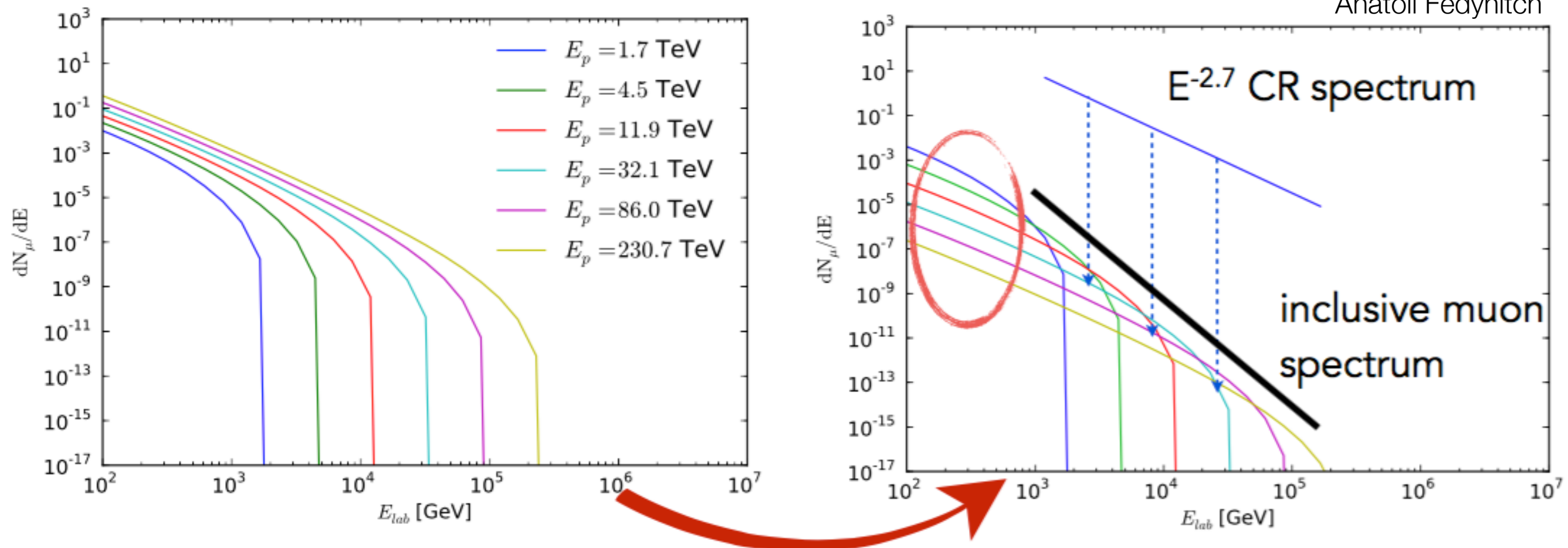


Anatoli Fedynitch

# inclusive spectrum

convolution with primary cosmic ray spectrum

Anatoli Fedynitch



$$\Phi_\mu(E_\mu) = \sum_p \sum_{E_i} w_p(E_i) \times Y_{p \rightarrow \mu}(E_\mu, E_i)$$

nuclei  
composition

primary spectrum

$\mu$  production yield  
e.g. from MC calculations

# inclusive lepton spectra

## interactions in the atmosphere

---

- inclusive lepton spectrum in the atmosphere

$$\phi_l(E_l) = \phi_N(E_l) \times \left\{ \frac{A_{\pi l}}{1 + B_{\pi l} \cos \theta E_l / \epsilon_\pi} + \frac{A_{Kl}}{1 + B_{Kl} \cos \theta E_l / \epsilon_K} + \dots \right\}$$

hadronization & cascading

$\phi_N(E) \approx E^{-(\gamma+1)}$

- spectrum-weighted Z-moments

$$A_{il} = \frac{Z_{Ni} \times BR_{il} \times Z_{il}}{1 - Z_{NN}} \quad \boxed{i = \pi, K}$$

- for inclusive  $\pi^+$  production

$$Z_{N\pi^+} = \frac{1}{\sigma_N^{air}} \int_0^1 x^\gamma \frac{d\sigma_{N\pi^+(x)}}{dx} dx \quad \gamma \approx 1.7$$

(Feynman scaling)

$$x \equiv \frac{p_{\parallel}}{p_{\parallel, max}} \sim \frac{2p_{\parallel}}{\sqrt{s}}$$

# spectrum-weighted moments

---

- energy-independent form

$$Z_{kh} = \int_0^1 x_{\text{LAB}}^{\gamma} \frac{dn_{k \rightarrow h}}{dx_{\text{LAB}}} dx_{\text{LAB}}$$

- energy-dependent with generic cosmic ray primary spectrum

$$Z_{kh}(E) = \int_E^{\infty} dE' \frac{d\Phi_N(E', \theta)}{d\Phi_N(E, \theta)} \frac{\sigma_{kA}(E')}{\sigma_{kA}(E)} \frac{dn_{k \rightarrow h}(E', E)}{dE}$$

# detecting neutrinos

## neutrino interactions with matter

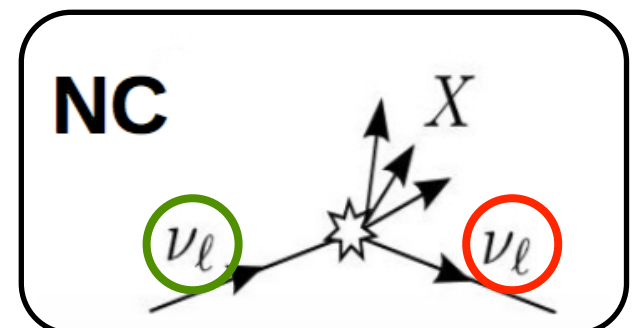
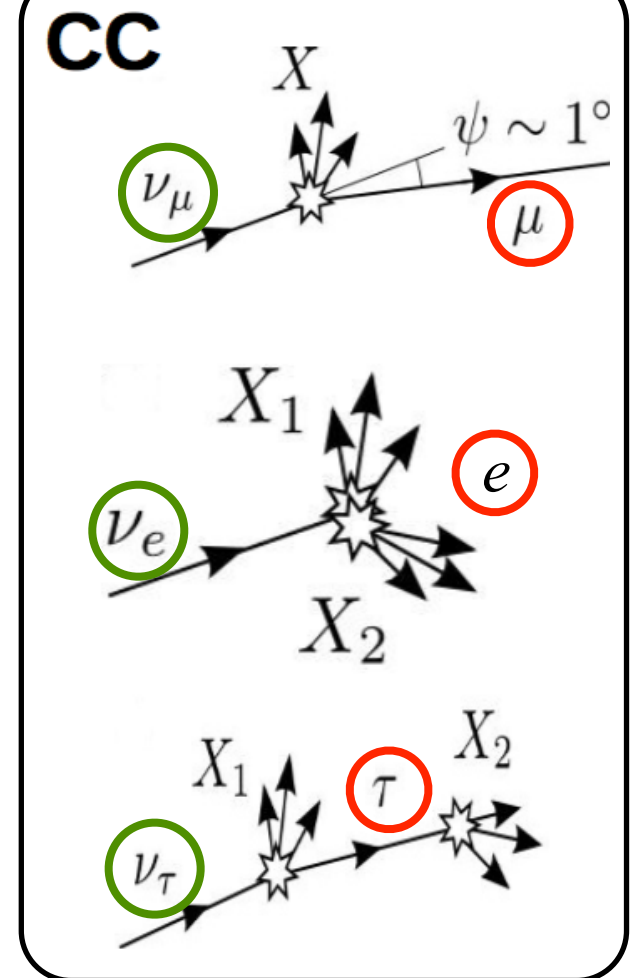
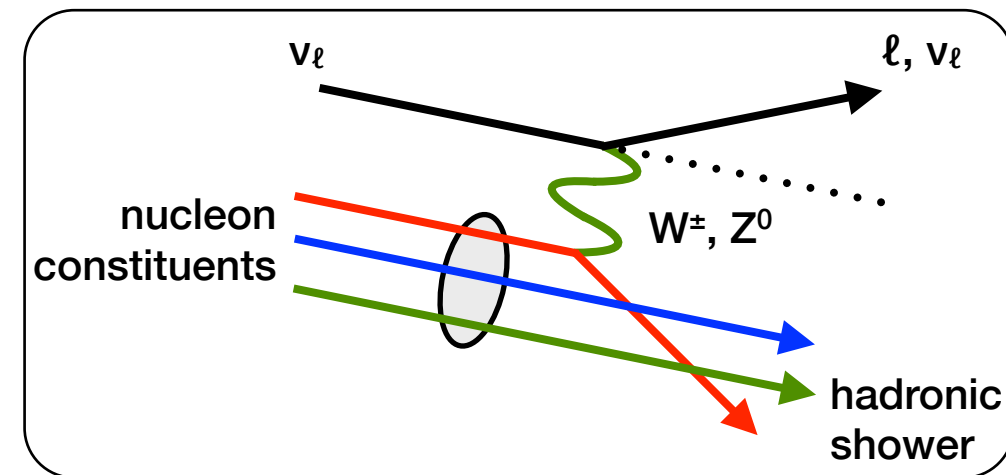
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- neutrino interacts with quark constituents of nucleons
- they exchange  $Z^0$  (**neutral**) or  $W^\pm$  (**charged**) bosons

► **charged current** interaction

► **neutral current** interaction

► **indirect detection of neutrinos**



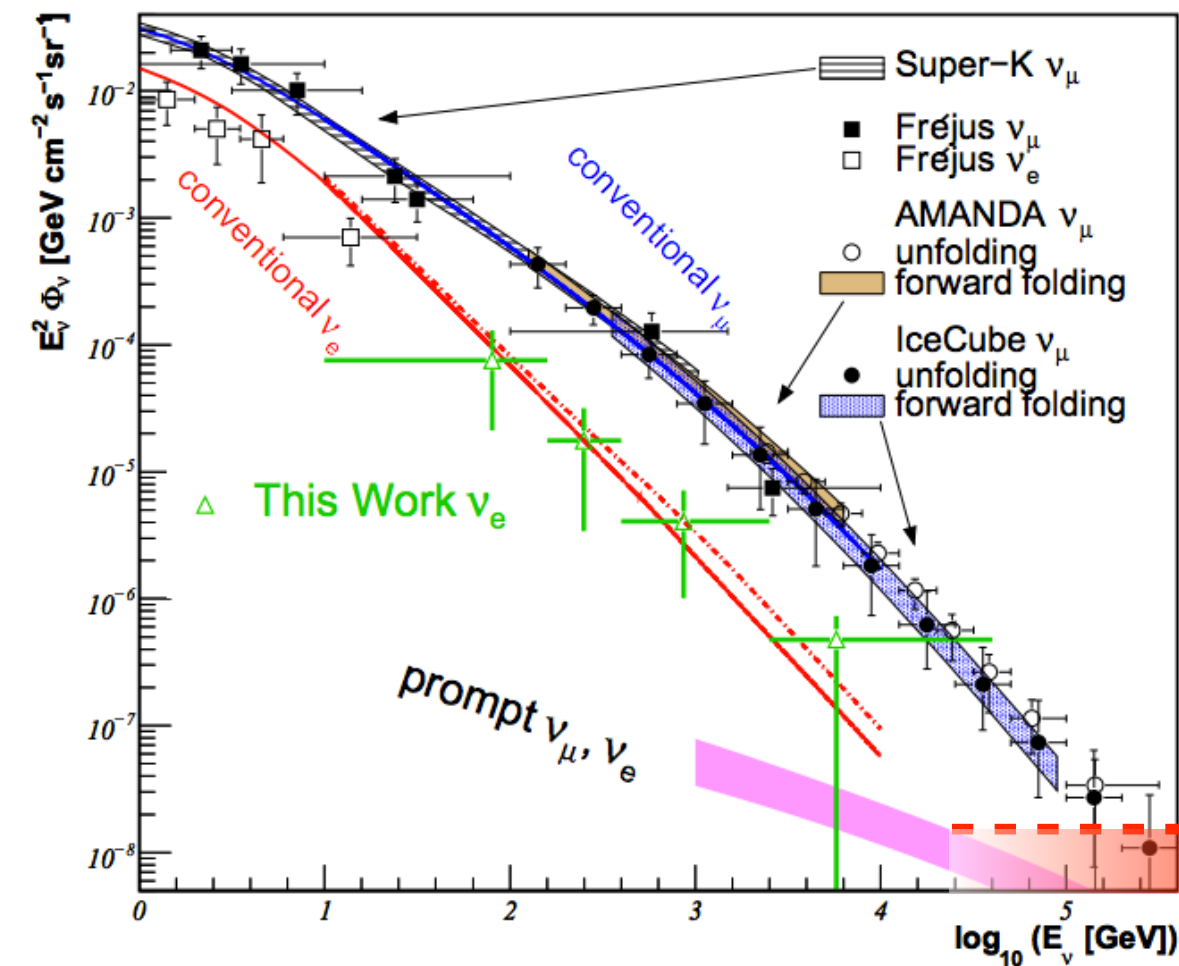


# atmospheric neutrinos

## high energy and heavy quarks

- ▶ **neutrino telescopes** searching for high energy astrophysical neutrinos (*point to origin of CR*)
- ▶ **atmospheric neutrinos** a significant **irreducible background** at high energy where **heavy quark processes** are involved
- ▶ production of **hyperons** and particles with **charm** affected by increasing uncertainties

$$\phi_\nu(E_\nu) = \phi_N(E_\nu) \times \left\{ \frac{A_{\pi\nu}}{1 + B_{\pi\nu} \cos \theta E_\nu / \epsilon_\pi} + \frac{A_{K\nu}}{1 + B_{K\nu} \cos \theta E_\nu / \epsilon_K} + \frac{A_{\text{charm}\nu}}{1 + B_{\text{charm}\nu} \cos \theta E_\nu / \epsilon_{\text{charm}}} \right\}$$



$$A_{i\nu} = \frac{Z_{Ni} \times BR_{i\nu} \times Z_{i\nu}}{1 - Z_{NN}}$$

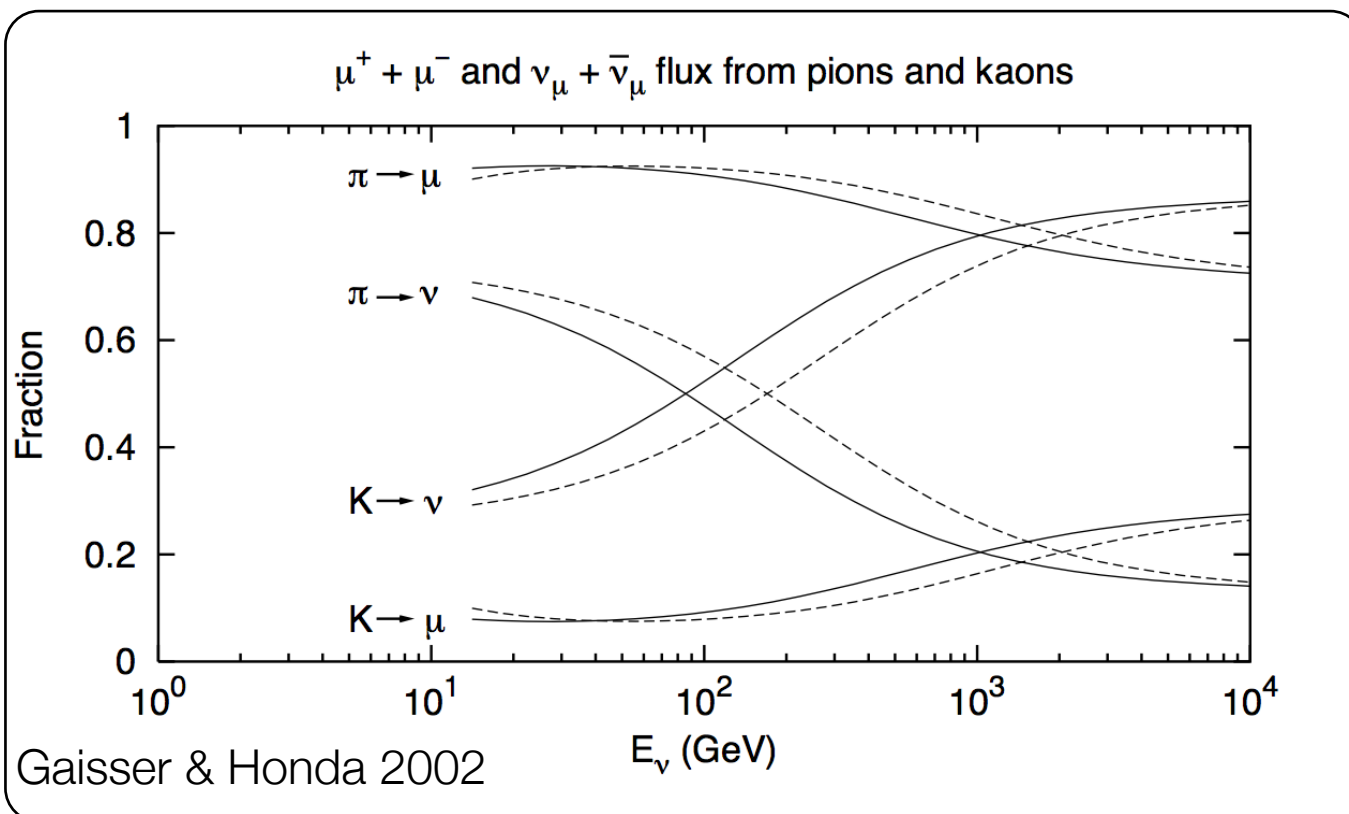
$$Z_{N\pi^\pm}(E) = \int_E^\infty dE' \frac{\phi_N(E')}{\phi_N(E)} \frac{\lambda_N(E)}{\lambda_N(E')} \frac{dn_{\pi^\pm}(E', E)}{dE}$$

meson's characteristic energy

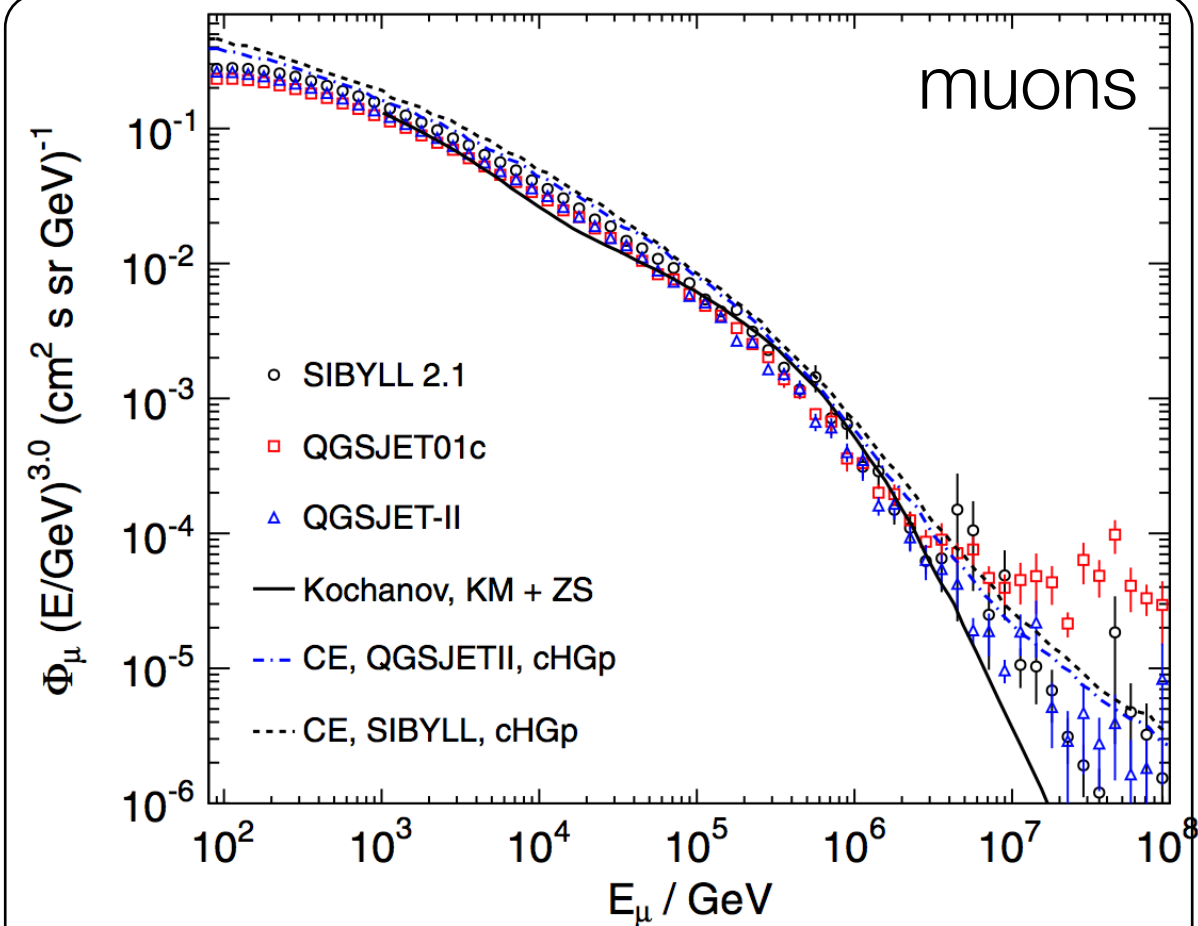
Particle ( $\alpha$ ):	$\pi^\pm$	$K^\pm$	$K_L^0$	Charm
$\epsilon_\alpha$ (GeV):	115	850	205	$\sim 3 \times 10^7$

# cosmic ray induced leptons

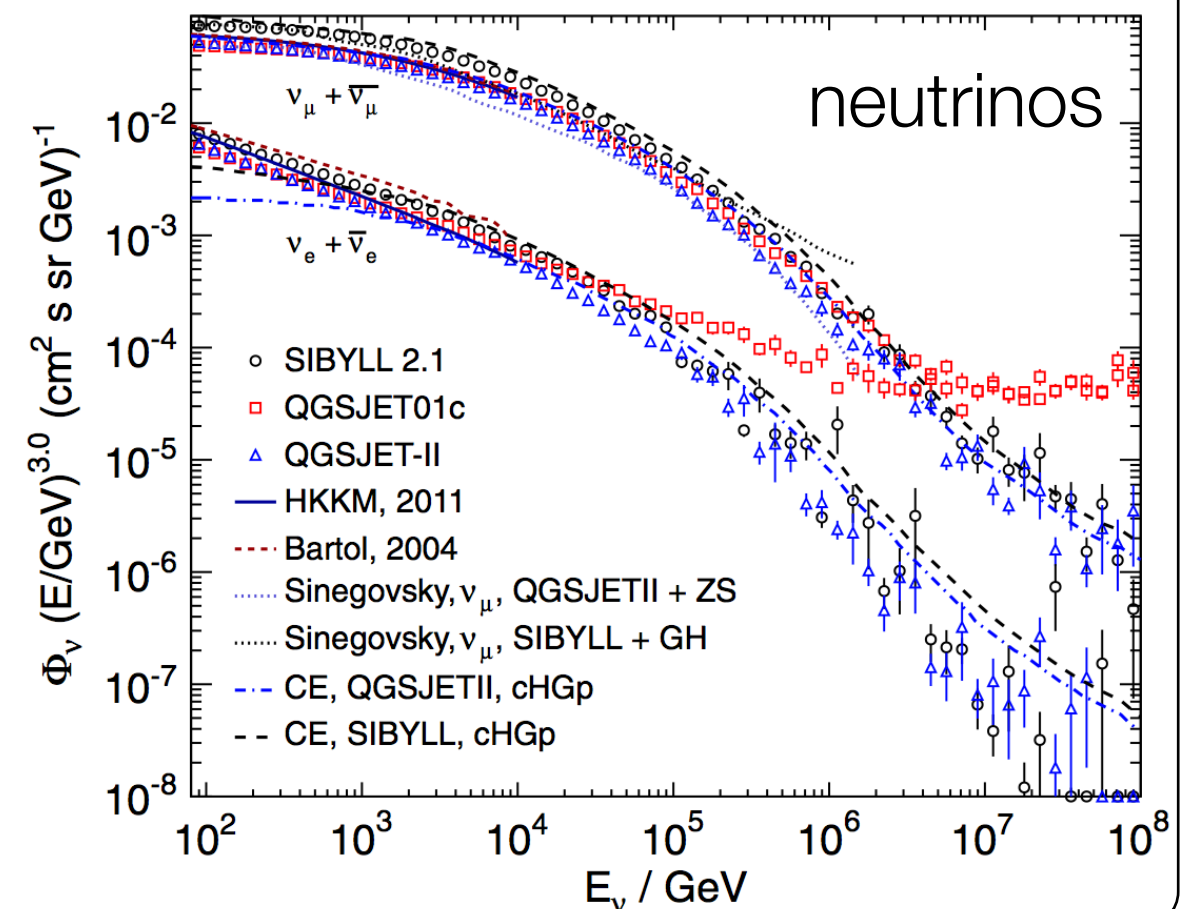
## interactions in the atmosphere



- kinematics: differences between  $\pi$  &  $K$
- $\nu_e$  neutrinos from (5%)  $K^\pm \rightarrow \pi^0 e^\pm \nu_e (\bar{\nu}_e)$   
and (41%)  $K_L^0 \rightarrow \pi + e + \nu_e$

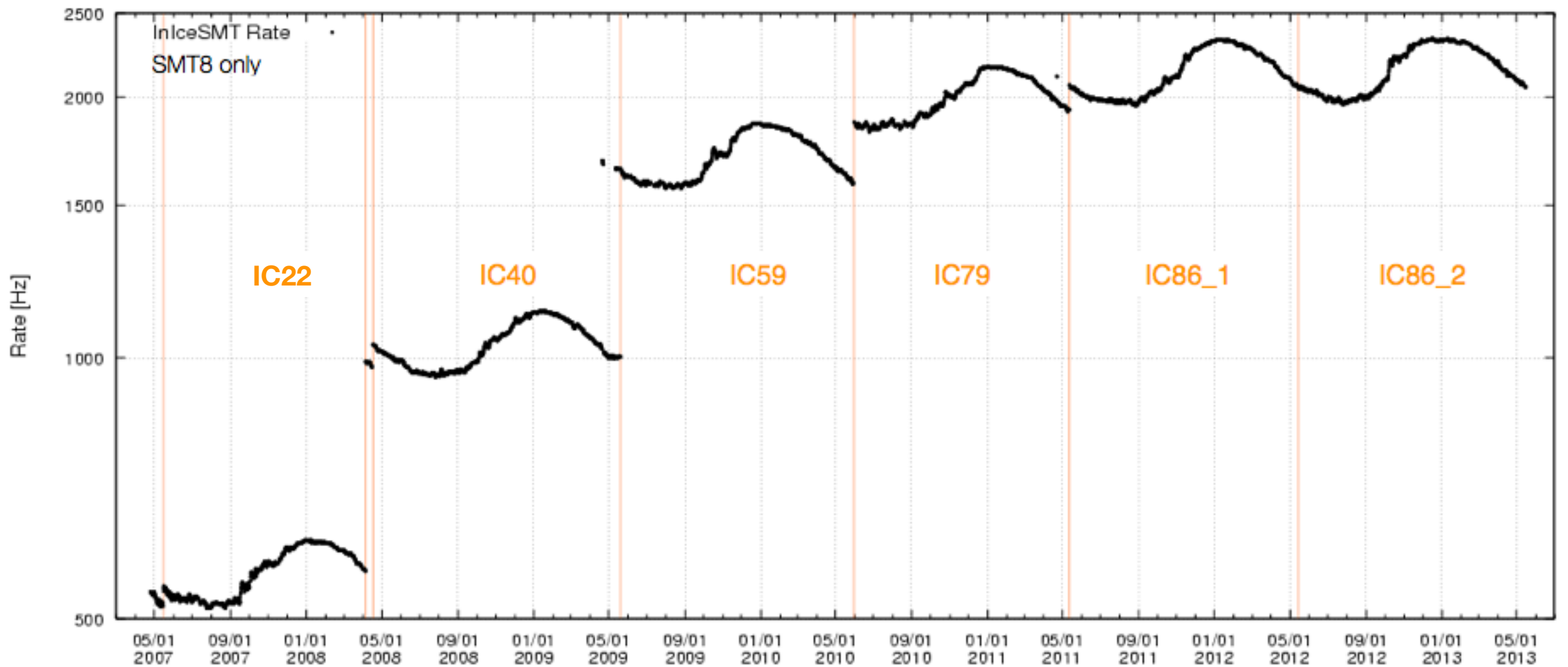


Fedynitch, Becker & PD 2013



# event rate in IceCube

## growing experiment



2007-08

2008-09

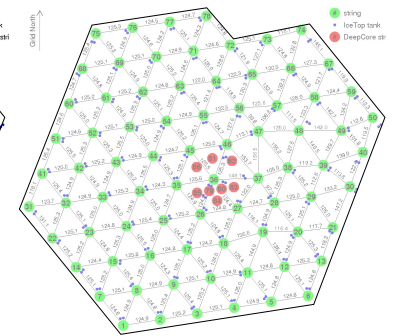
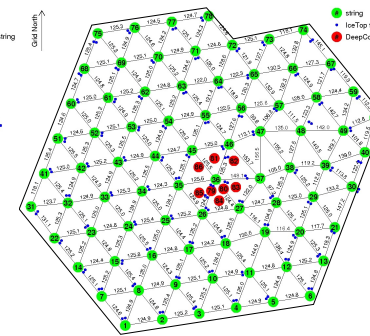
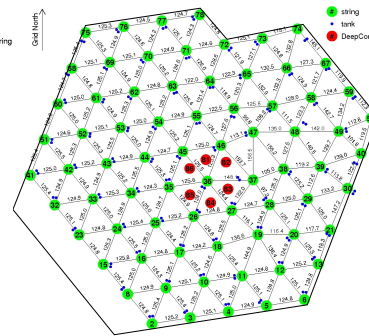
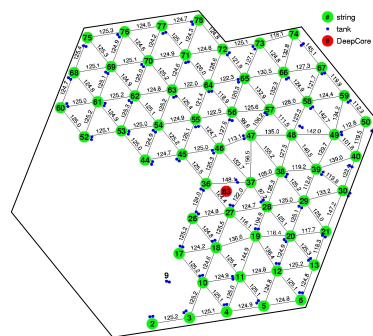
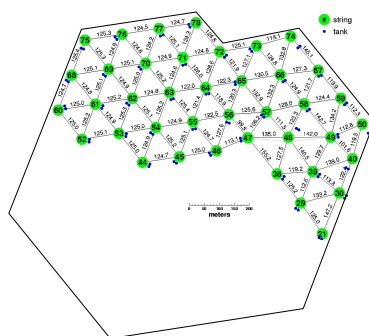
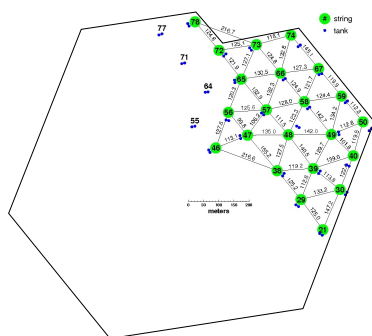
2009-10

2010-11

2011-12

2012-13

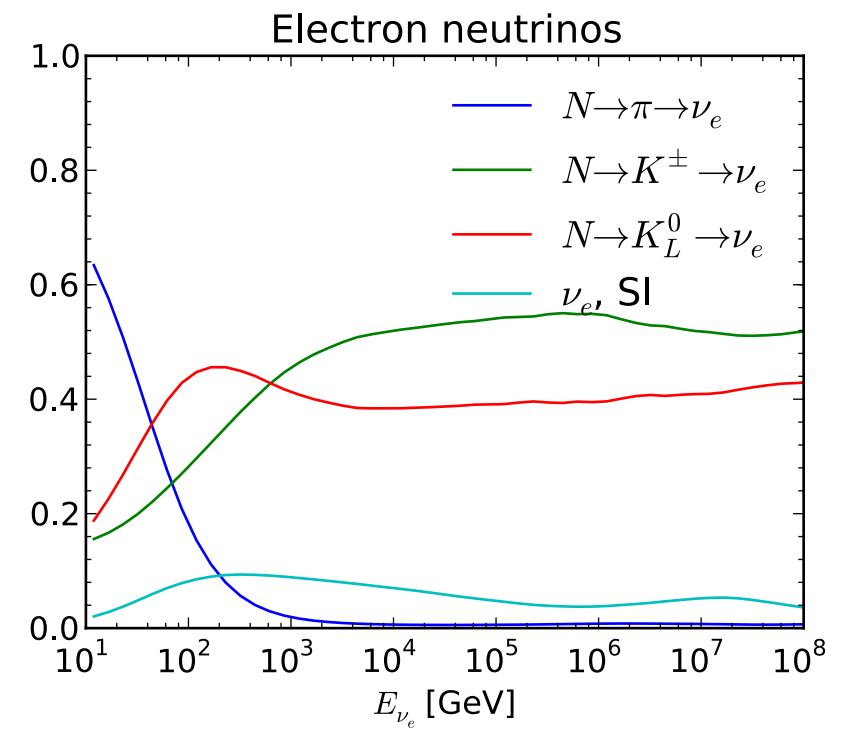
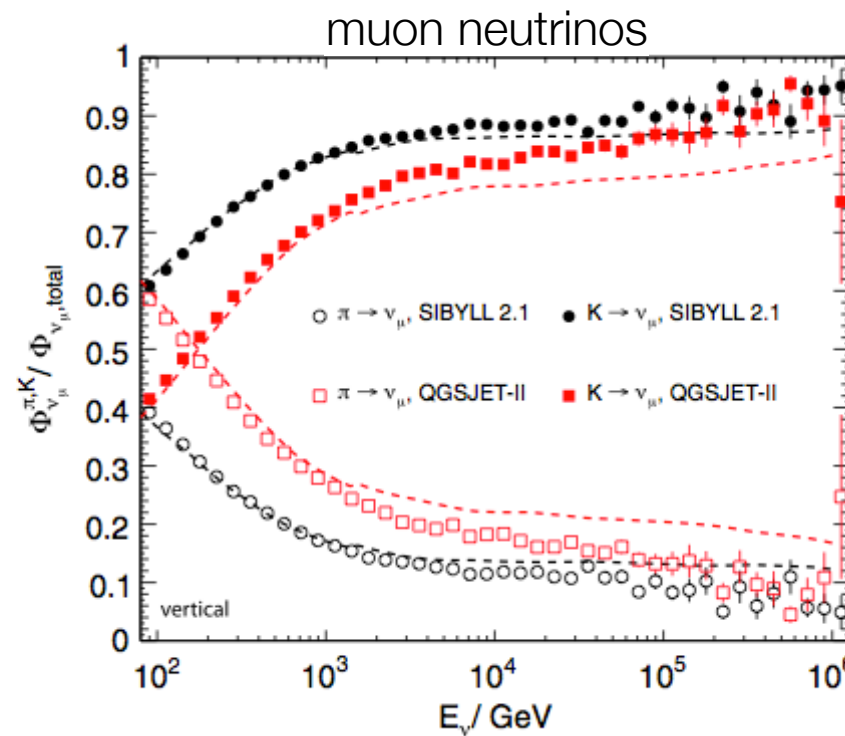
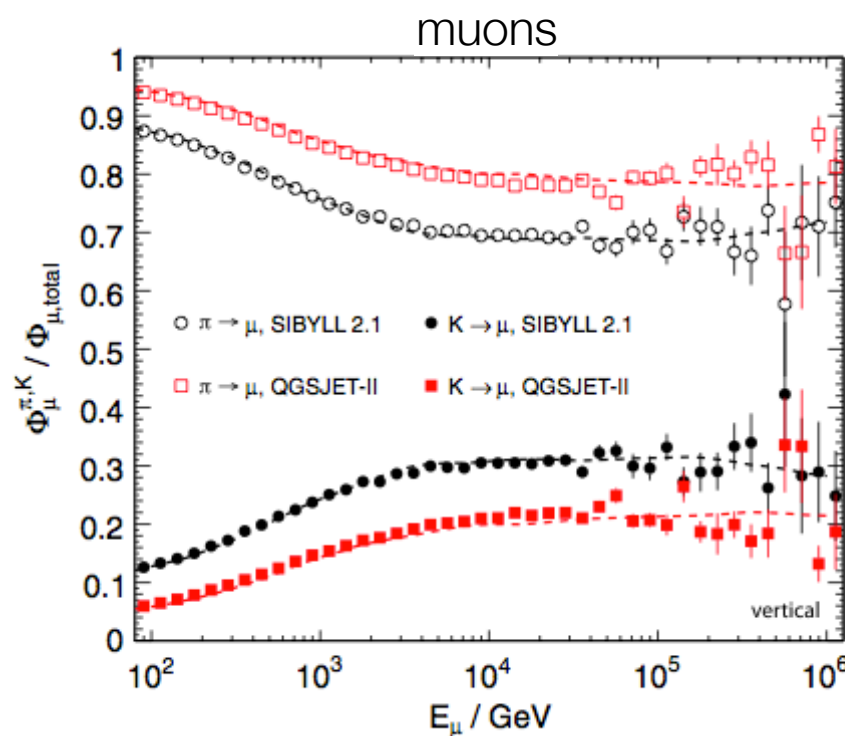
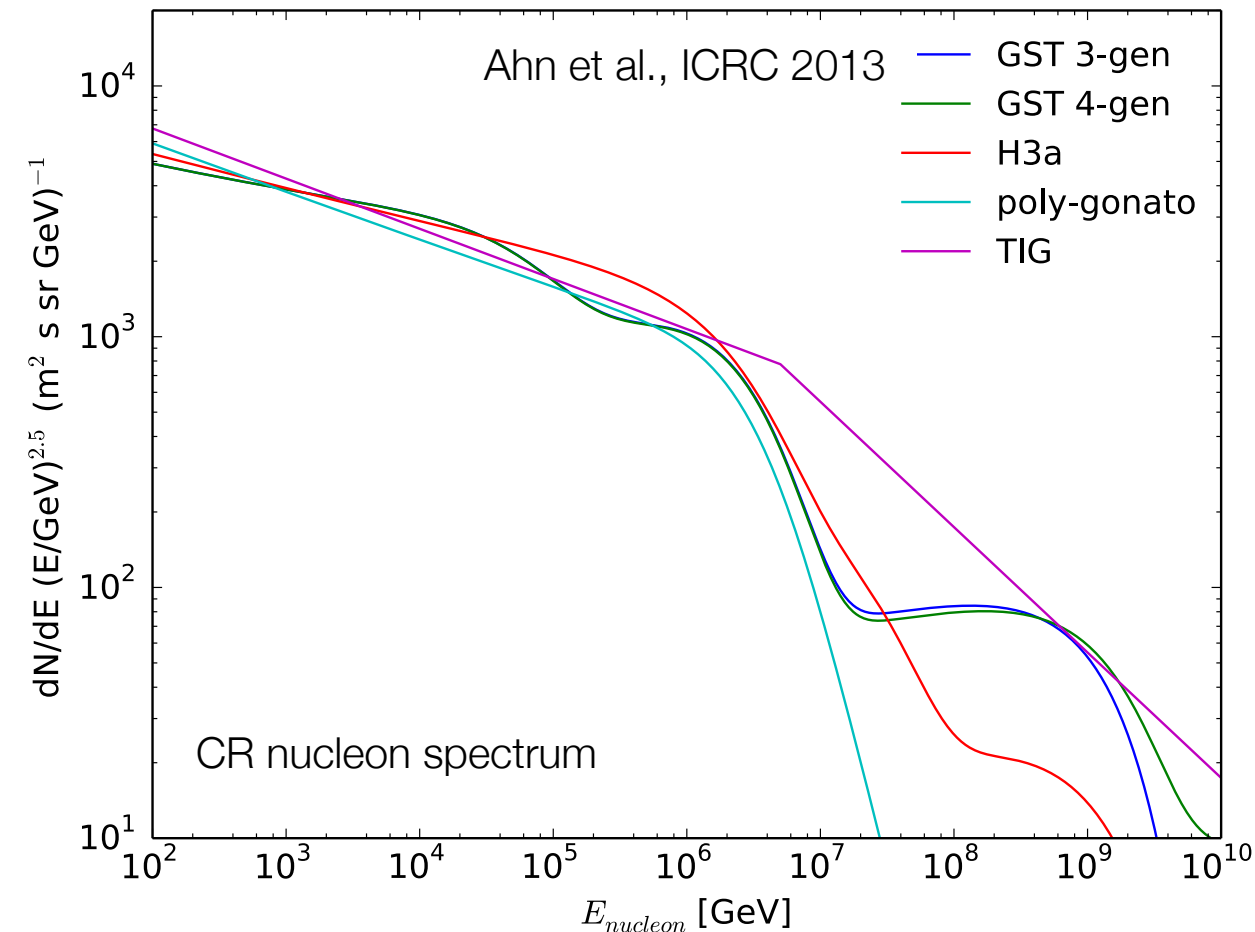
>>



# atmospheric neutrinos

## high energy and heavy quarks

- ▶ large uncertainties in **cosmic ray composition** (nucleon spectrum) at high energy
- ▶  $K^\pm$  not same isospin group & K evolution equations coupled
- ▶ **associated production**  $p + \text{Air} \rightarrow \Lambda + K^+$



Fedynitch, Becker Tjus, PD 2012

Sibyll 2.1 - Fedynitch

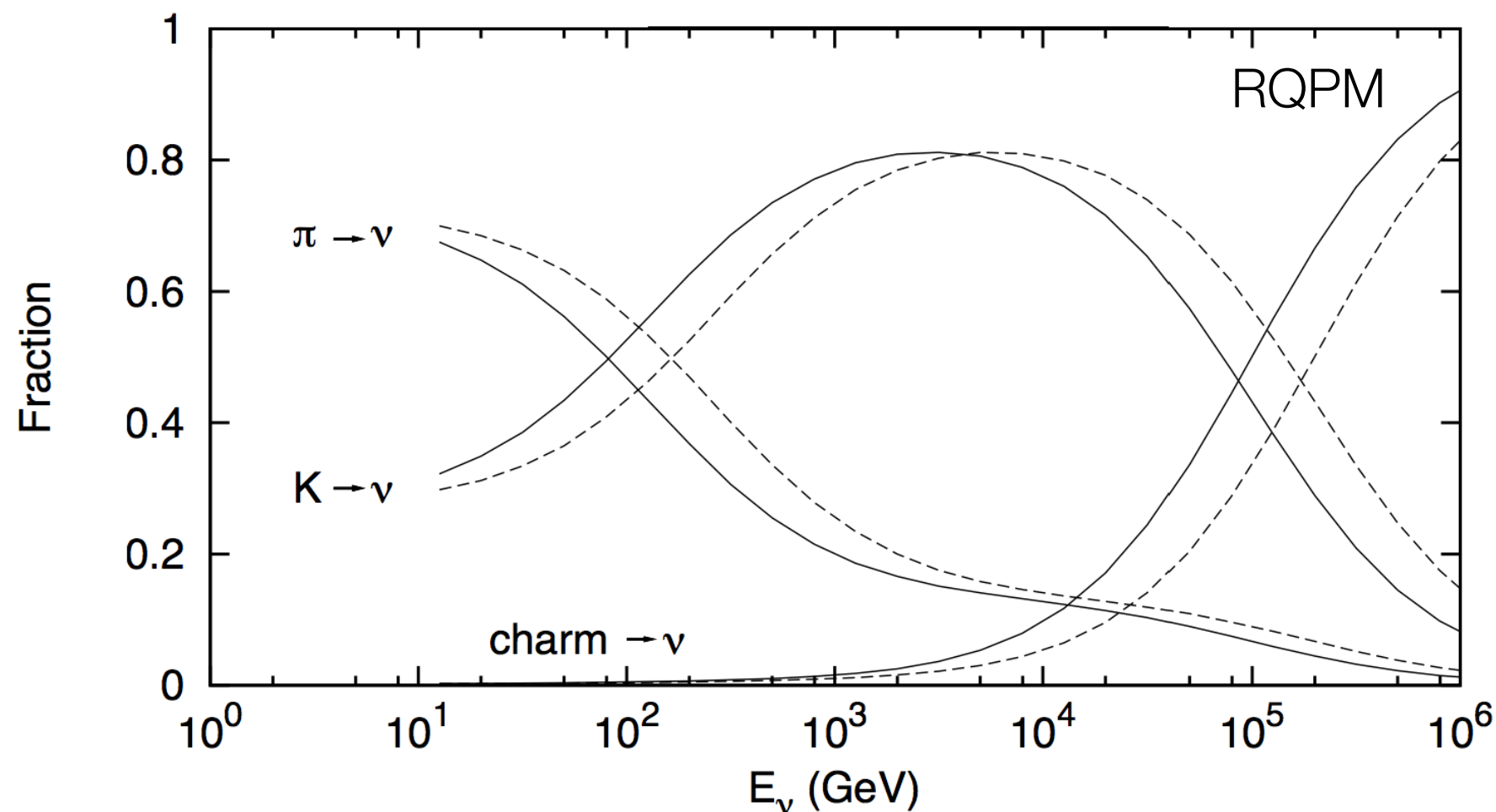


# heavy quark production in the atmosphere

## charm and prompt component

- **D** mesons and  $\Lambda_c^+$  baryons (with charm quark) decay “**promptly**”

$$\phi_l(E_l) = \phi_N(E_l) \times \left\{ \frac{A_{\pi l}}{1 + B_{\pi l} \cos \theta E_l / \epsilon_\pi} + \frac{A_{Kl}}{1 + B_{Kl} \cos \theta E_l / \epsilon_K} + \frac{A_{charm l}}{1 + B_{charm l} \cos \theta E_l / \epsilon_{charm}} \right\}$$



$$\tau_D \approx 10^{-12} sec$$

$$\tau_{\Lambda_c^+} \approx 10^{-13} sec$$

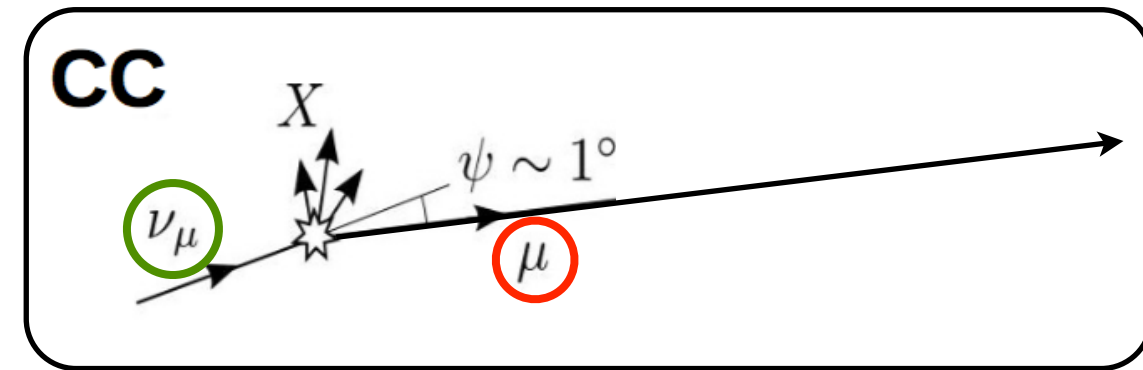
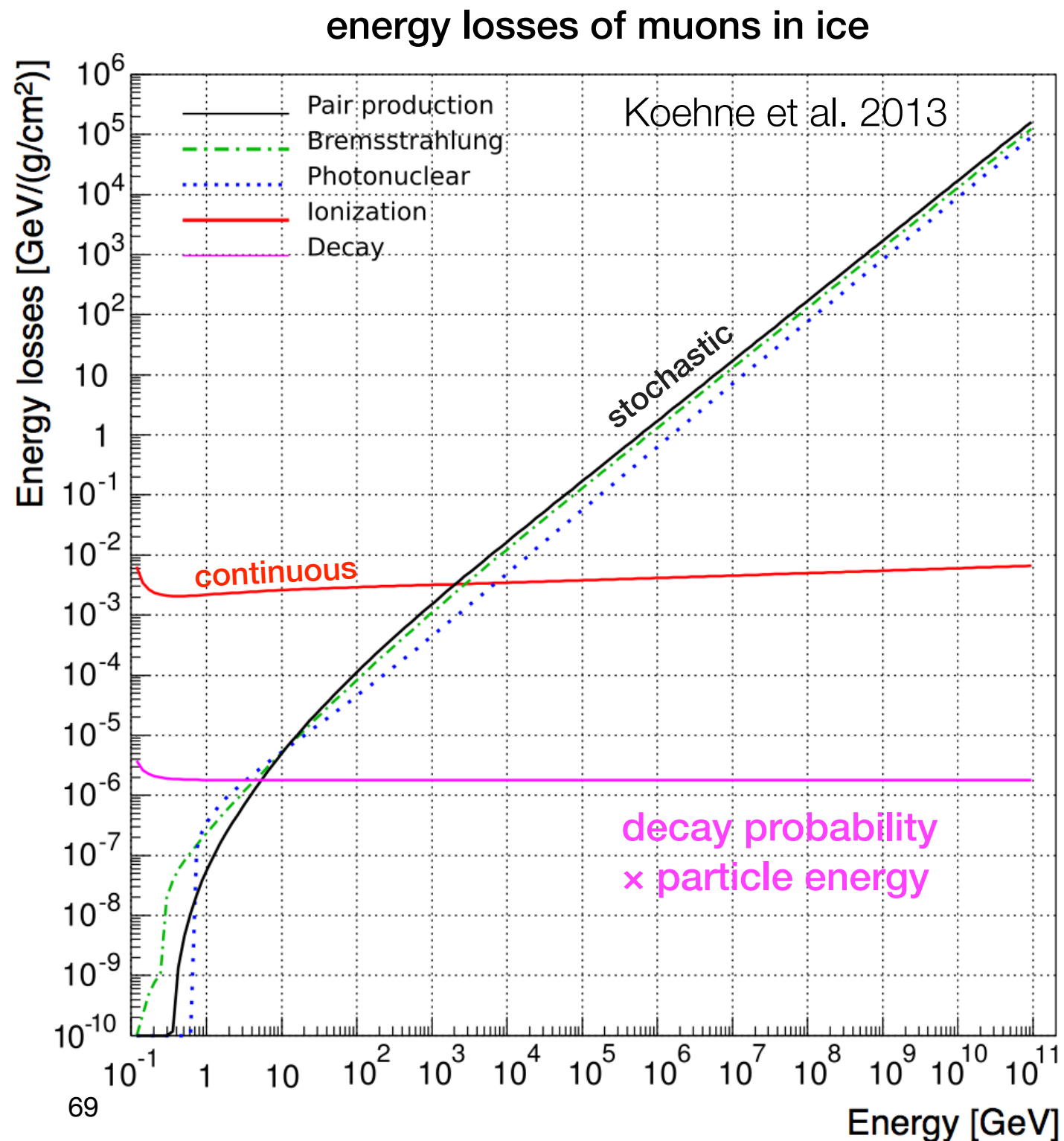
$$\epsilon_{D^0} = 9.2 \times 10^7 GeV$$

$$\epsilon_{D^\pm} = 4.3 \times 10^7 GeV$$



# detecting neutrinos

## detection of charged leptons



► indirect detection of  $\nu_\mu$  neutrinos

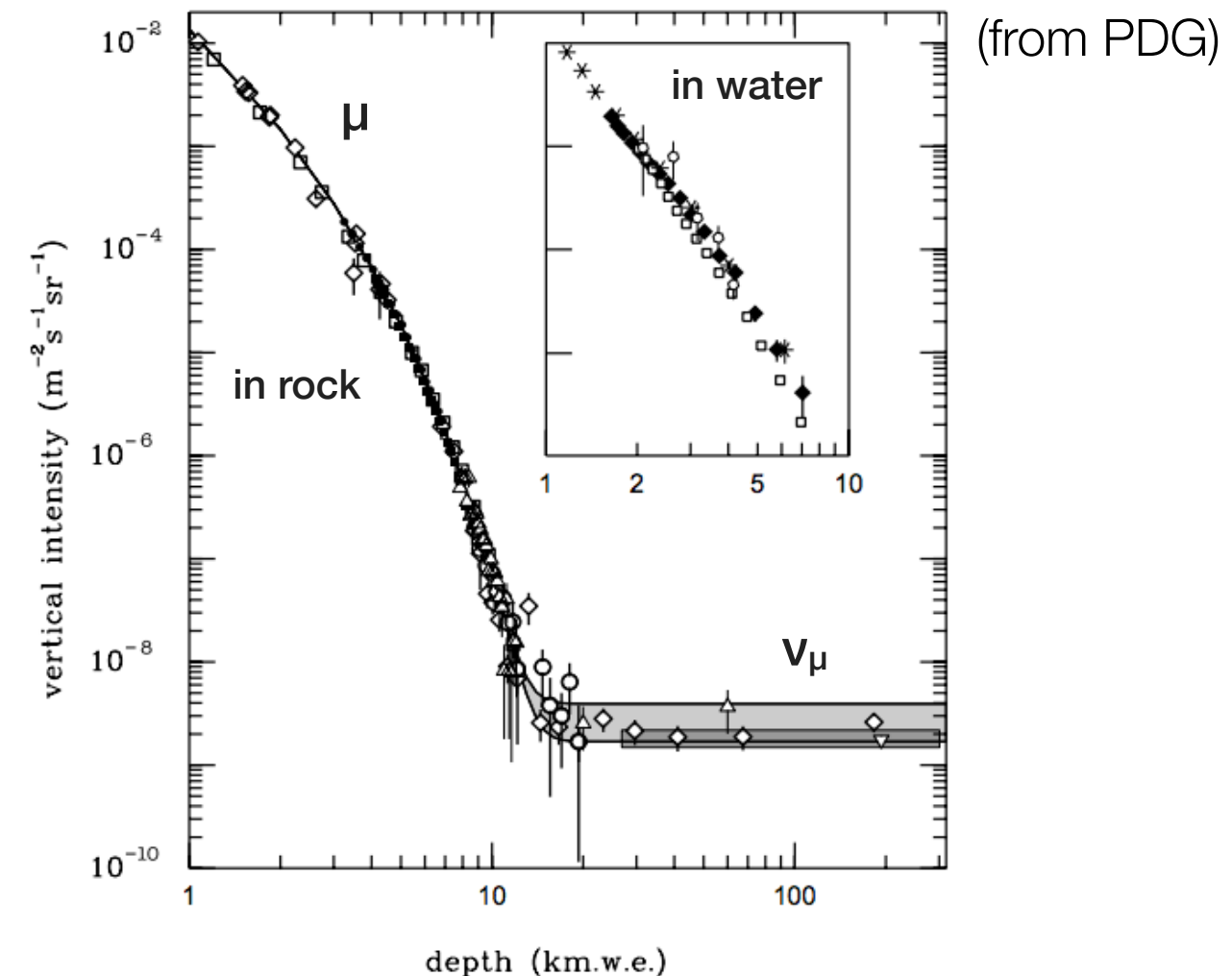
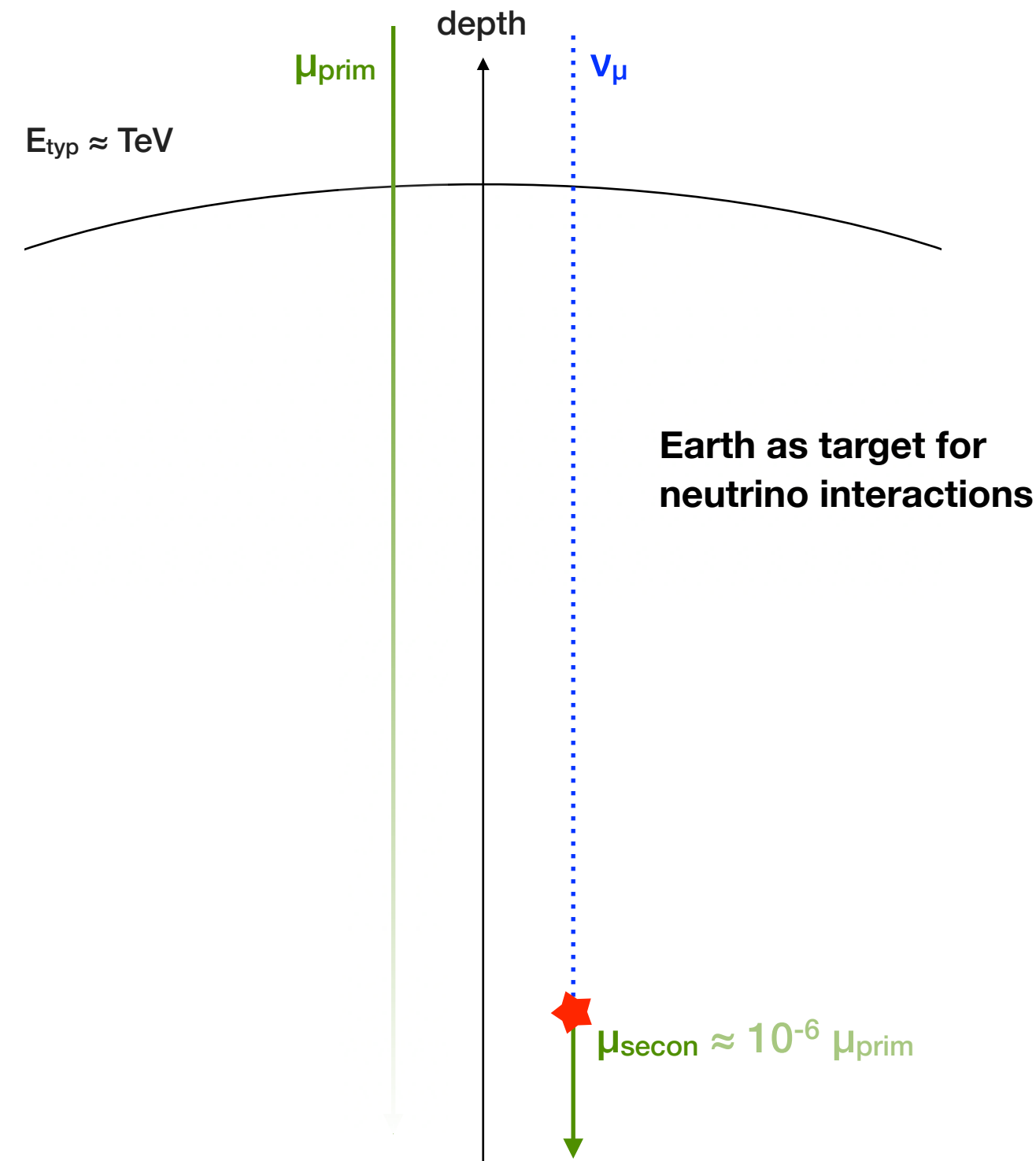
$$-\frac{dE_\mu}{dX} = a(E_\mu) + b(E_\mu) E_\mu$$

$$a = a_{ionization}$$

$$b = b_{brems} + b_{pair} + b_{nucl}$$

# detecting neutrinos

## detection of charged leptons

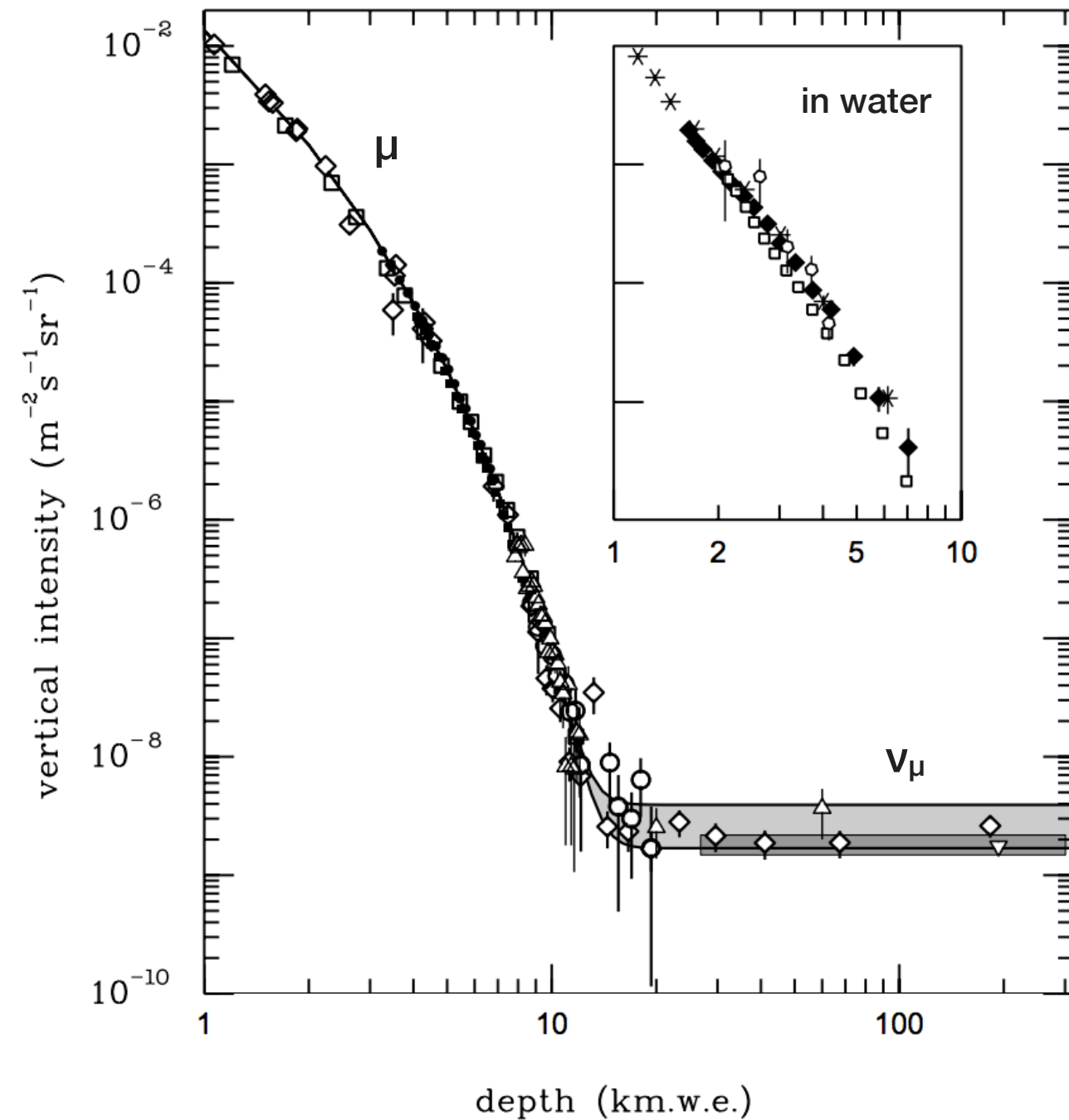
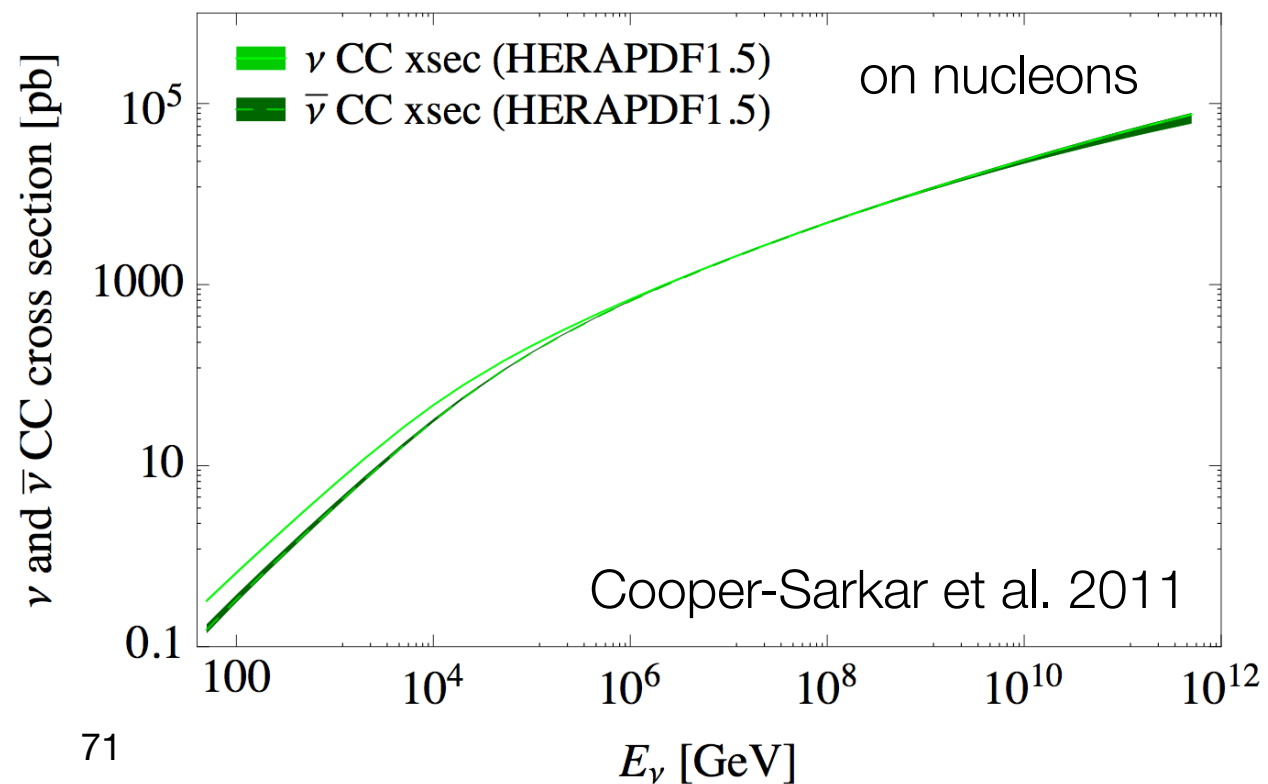


**Figure 27.7:** Vertical muon intensity vs depth ( $1 \text{ km.w.e.} = 10^5 \text{ g cm}^{-2}$  of standard rock). The experimental data are from:  $\diamond$ : the compilations of Crouch [67],  $\square$ : Baksan [72],  $\circ$ : LVD [73],  $\bullet$ : MACRO [74],  $\blacksquare$ : Frejus [75], and  $\triangle$ : SNO [76]. The shaded area at large depths represents neutrino-induced muons of energy above 2 GeV. The upper line is for horizontal neutrino-induced muons, the lower one for vertically upward muons. Darker shading shows the muon flux measured by the SuperKamiokande experiment. The inset shows the vertical intensity curve for water and ice published in Refs. [68–71].

# primary cosmic rays

## underground detection

- muons are penetrating but they lose energy within 10-20 km
- at greater depths experiments observe neutrino-induced muons
- $\nu_\mu(\bar{\nu}_\mu) + N \rightarrow \mu^\pm + X$  (CC interaction)



$$\Phi_\mu(E_\mu) \approx \Phi_\nu(E_\nu) \times P_{\nu\mu}(E_\nu, E_\mu)$$

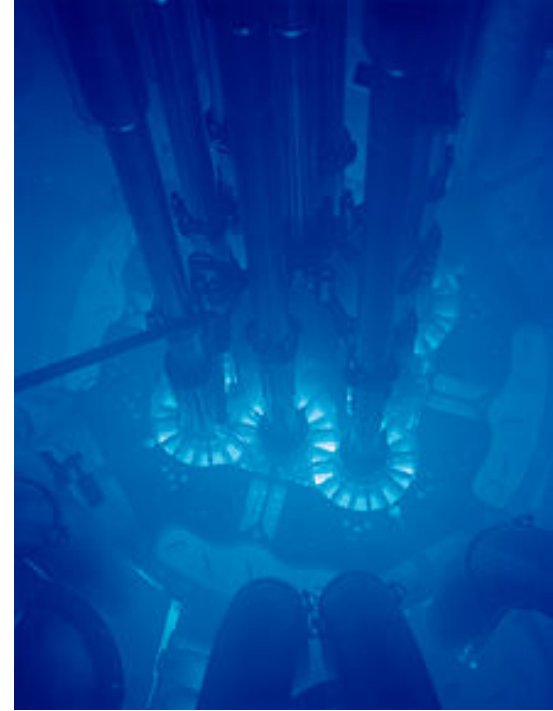
$$P_{\nu\mu}(E_\nu, E_\mu) \approx \frac{N_A}{A_{ice}} \times \sigma_{\nu\mu}^{CC}(E_\nu) \times R_\mu(E_\mu)$$

# detecting neutrinos in transparent media

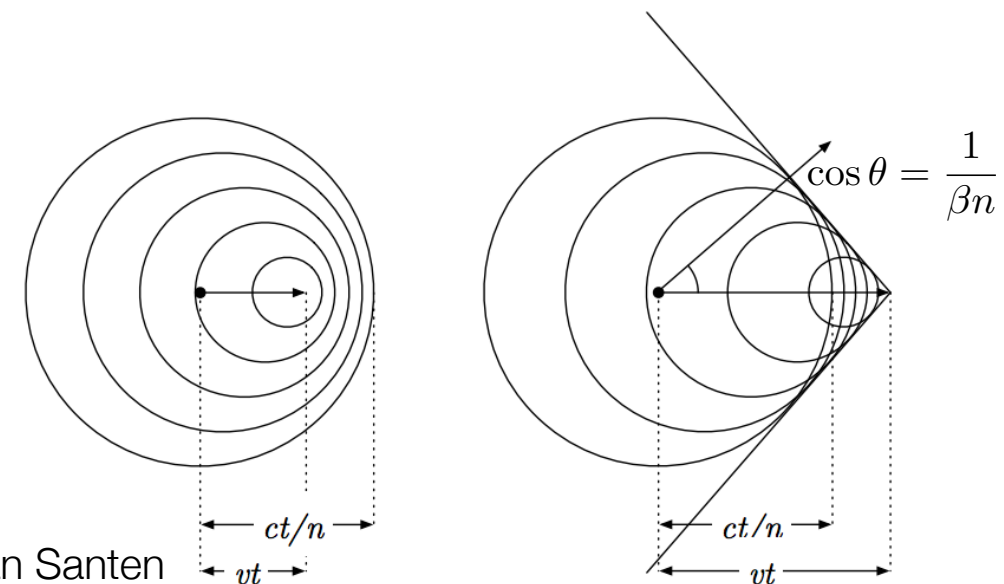
## Cherenkov Effect

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- electromagnetic radiation emitted when a charged particle passes through a **dielectric** medium at a speed greater than the phase velocity of light in that medium ( $v > c/n_{\text{medium}}$ )
- atoms near the particle become polarized and emit coherent radiation when returning to equilibrium



$$\frac{dN^2}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2} \left( 1 - \frac{1}{\beta^2 n^2} \right)$$



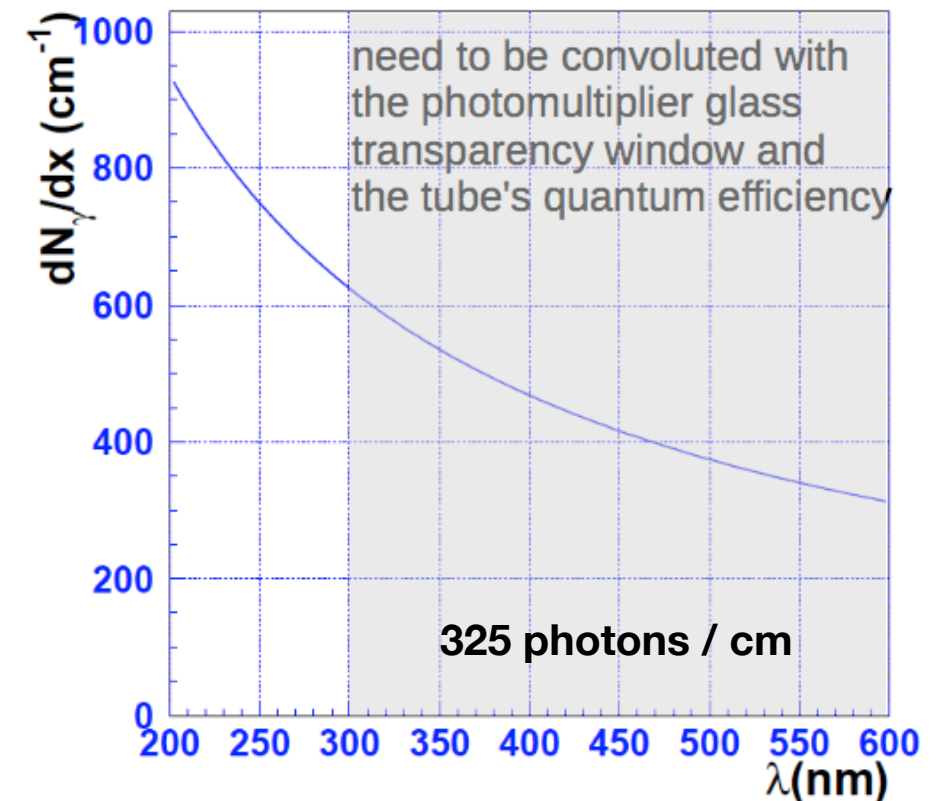
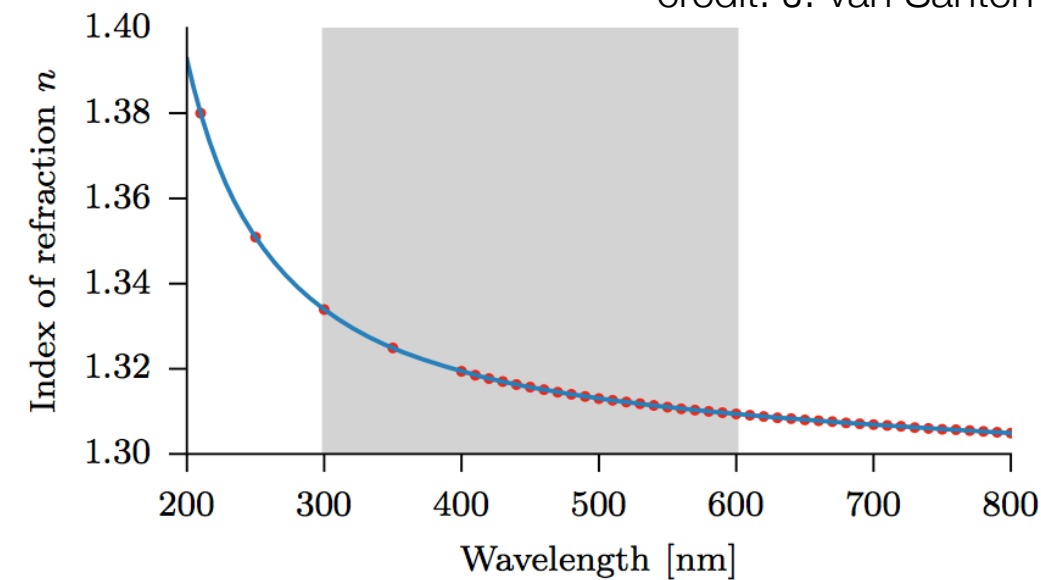
# detecting neutrinos in transparent media

## Cherenkov Effect

$$\frac{dN^2}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2} \left( 1 - \frac{1}{\beta^2 n^2} \right)$$

$$\frac{dN}{dx} = 2\pi\alpha \sin^2 \theta \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)$$

credit: J. van Santen

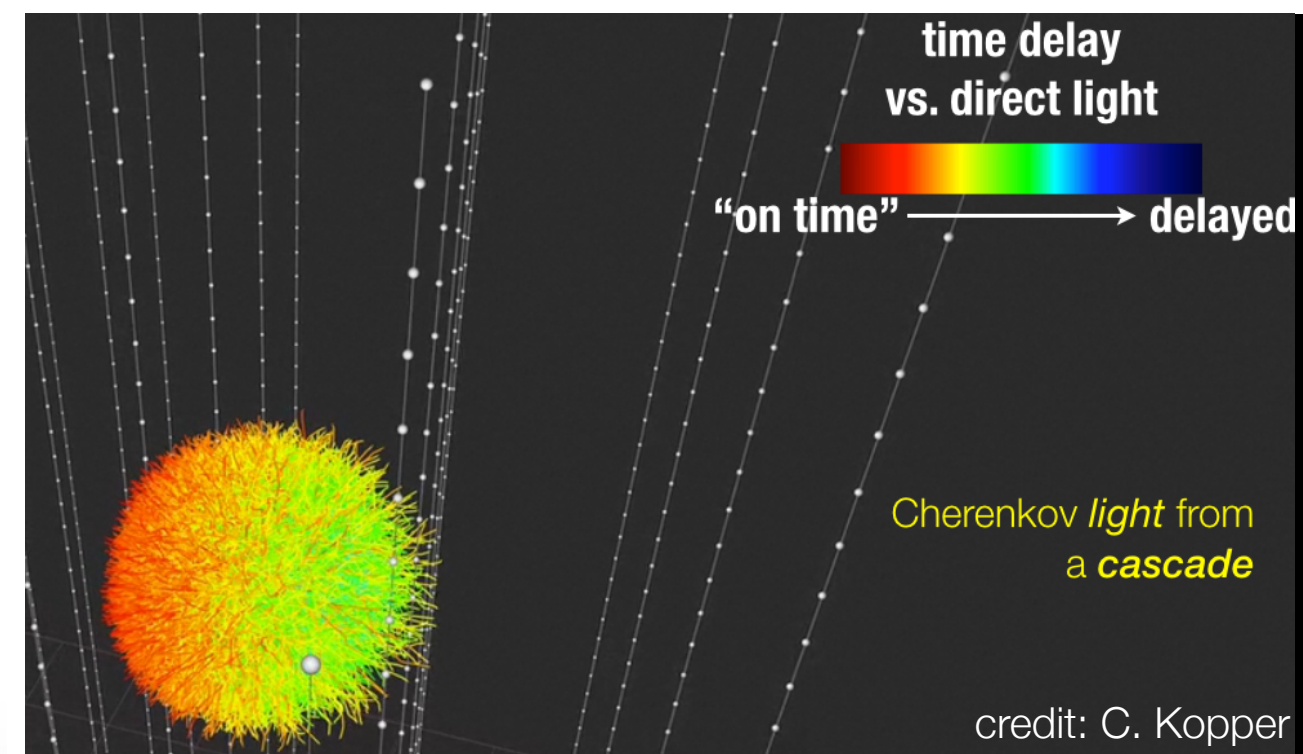
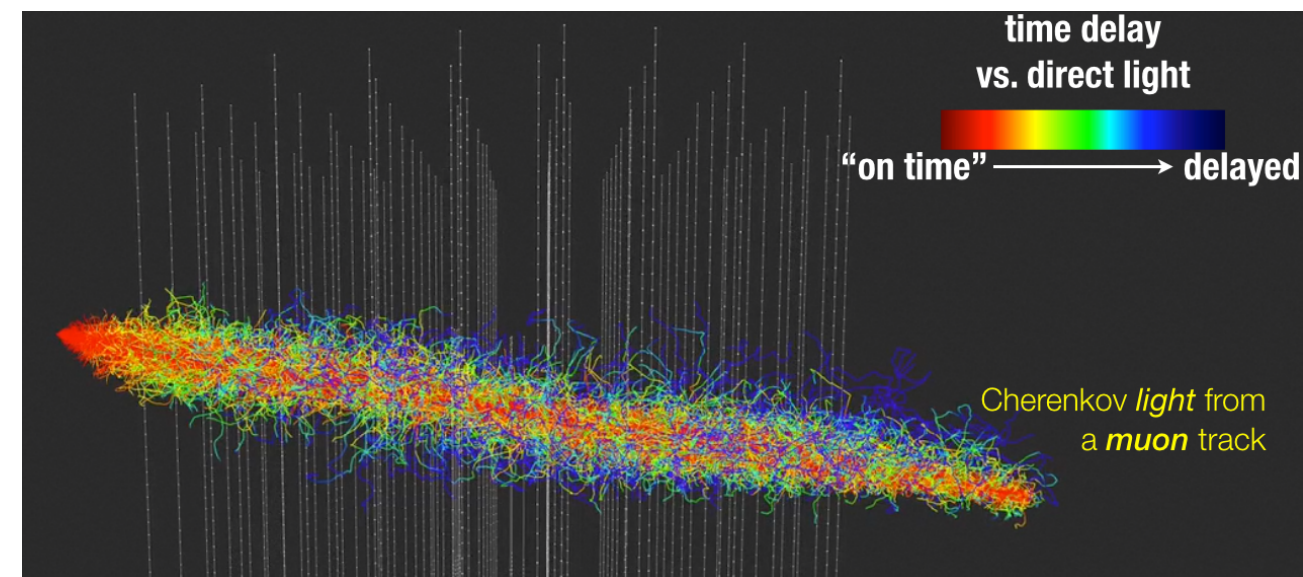
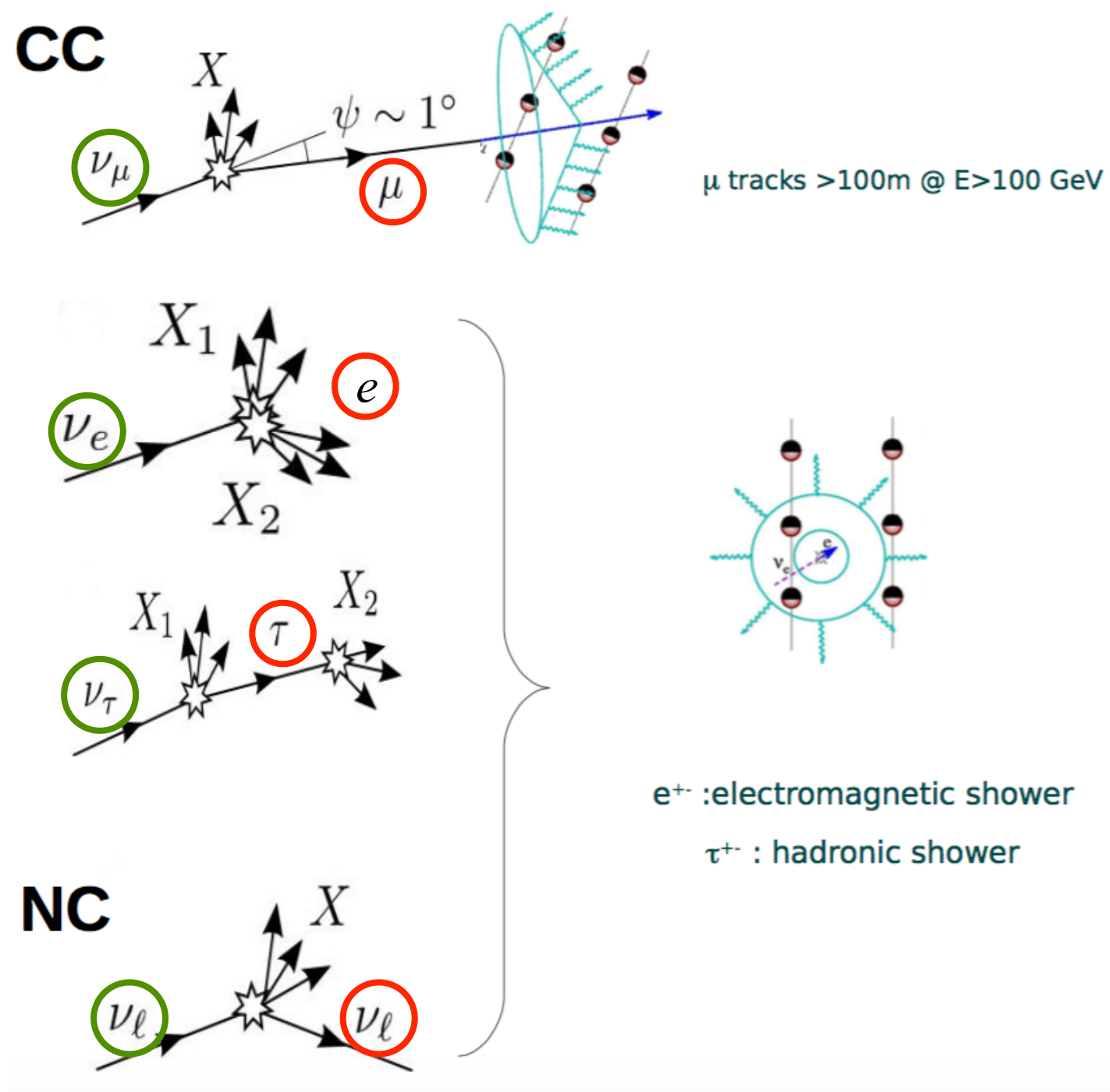




# detection technique

## Cherenkov radiation

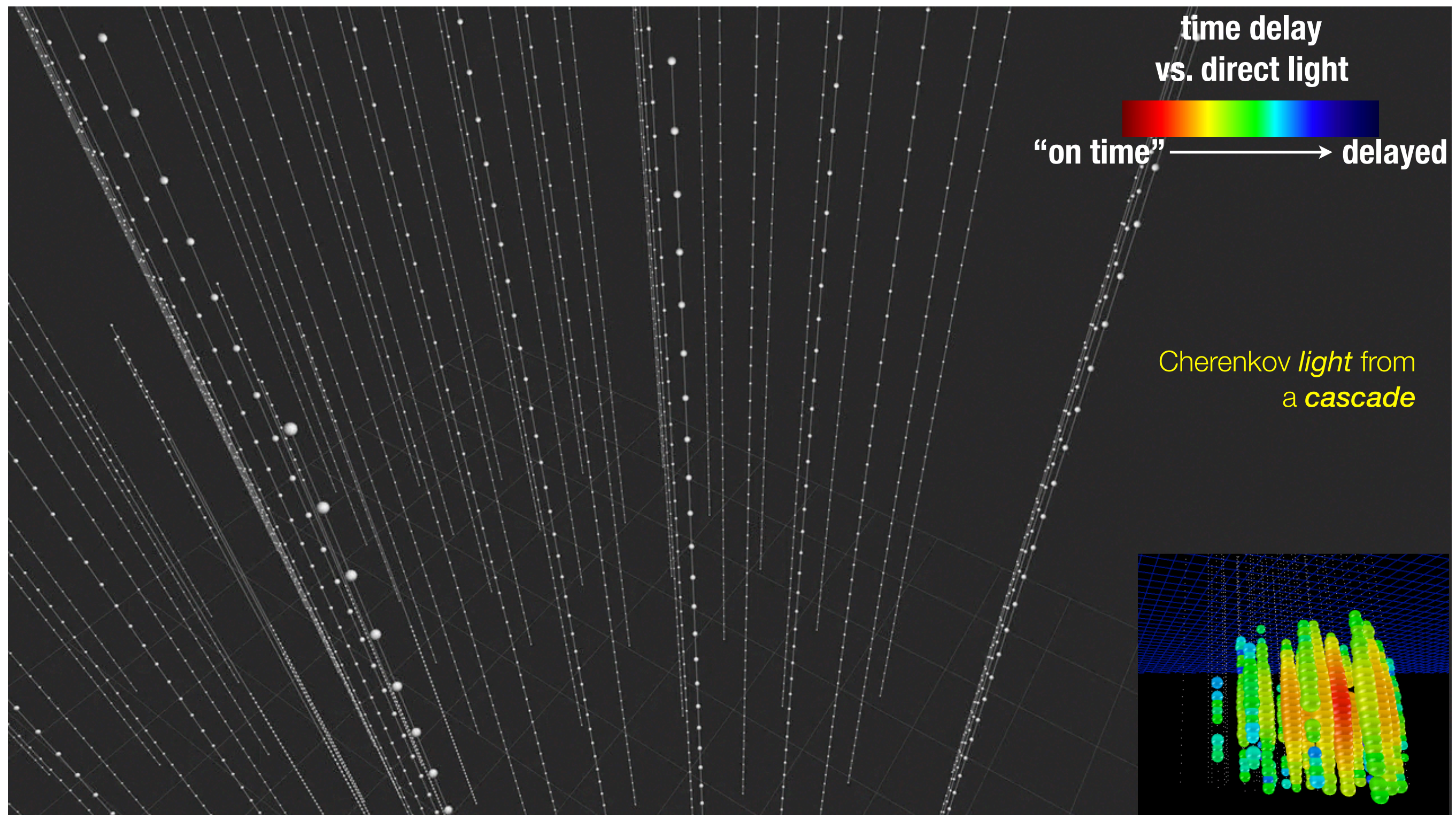
photon scattering  
in the transparent medium





# detection principle - *cascade*

$\nu_e$   $\nu_\tau$  CC-int &  $\nu_i$  NC-int



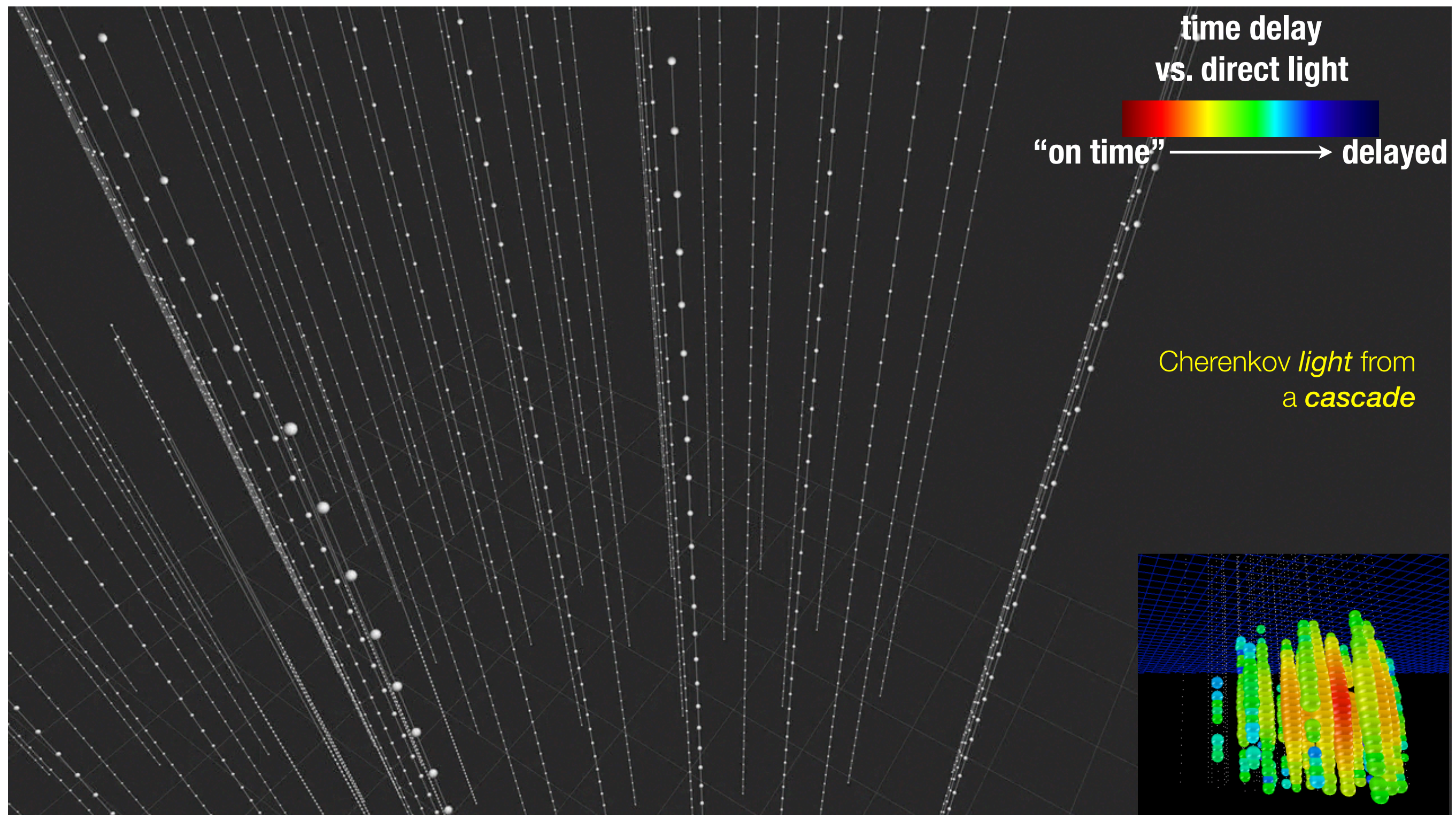
$\approx \pm 15\%$  deposited energy resolution  
 $\approx 10^\circ$  angular resolution  
(at energies  $\gtrsim 100\text{TeV}$ )

Claudio Kopper - WIPAC



# detection principle - *cascade*

$\nu_e$   $\nu_\tau$  CC-int &  $\nu_i$  NC-int



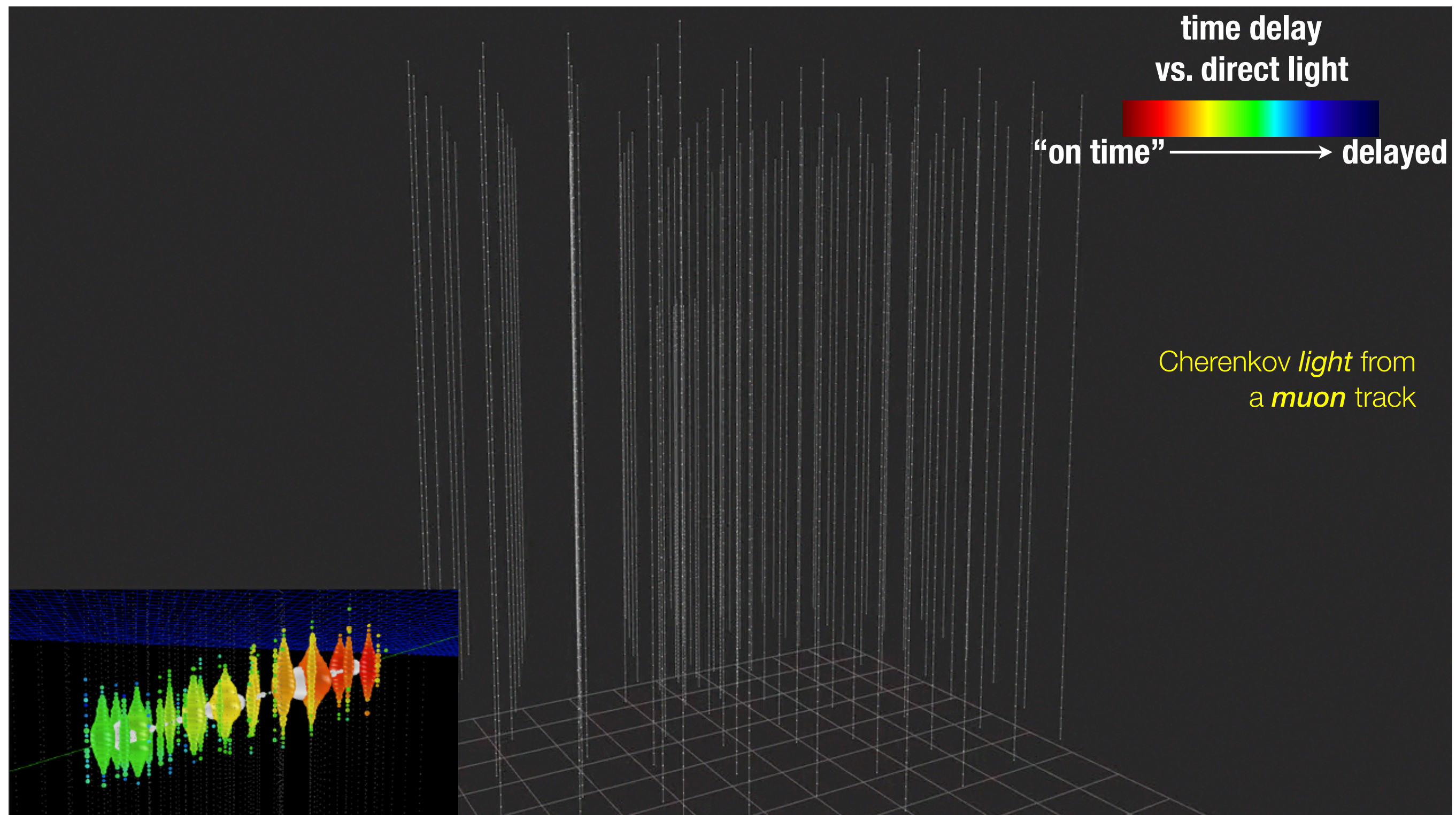
$\approx \pm 15\%$  deposited energy resolution  
 $\approx 10^\circ$  angular resolution  
(at energies  $\gtrsim 100\text{TeV}$ )

Claudio Kopper - WIPAC



# detection principle - *track*

$\nu_\mu$  CC-int



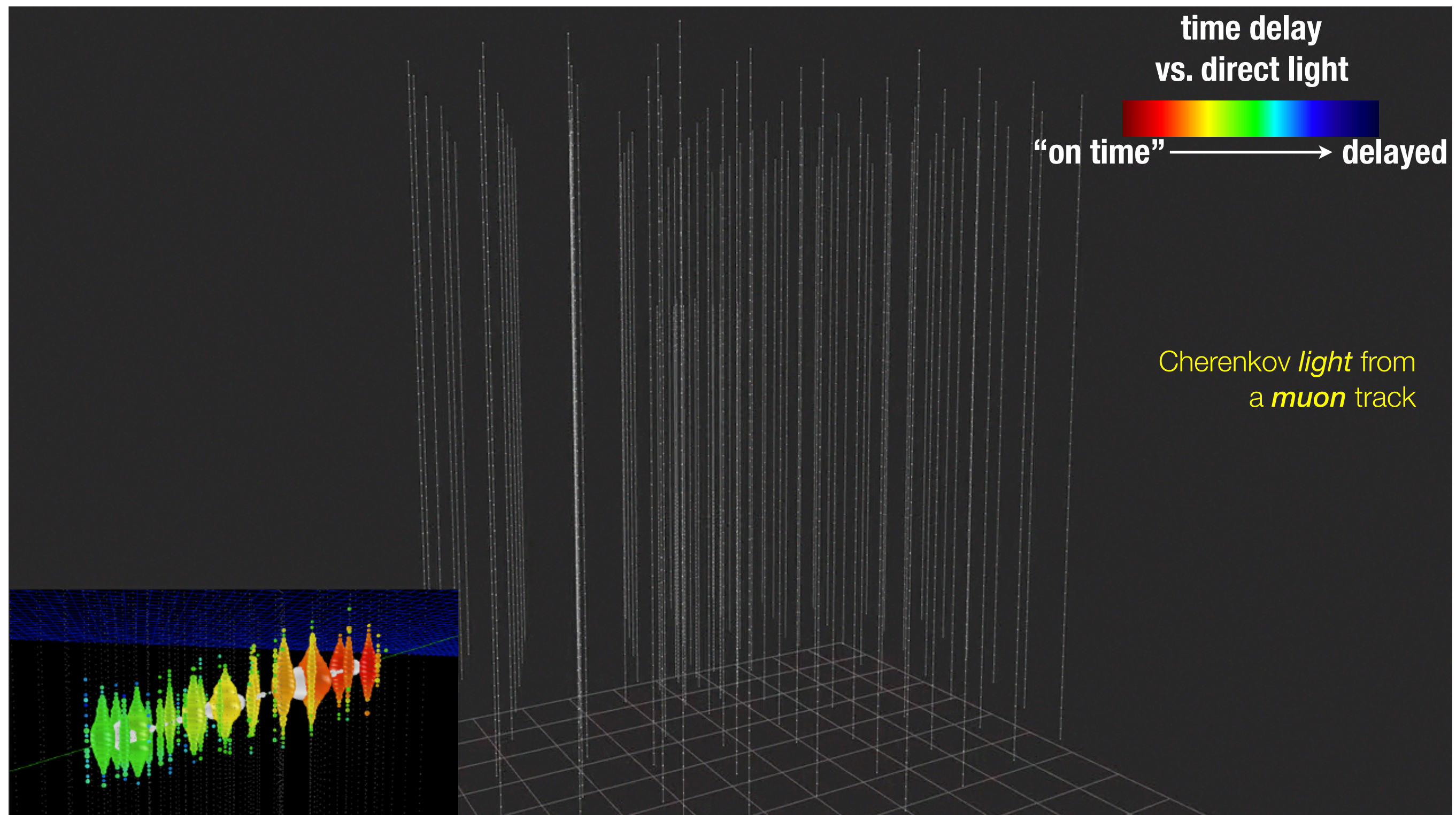
factor of  $\approx 2$  energy resolution  
<  $1^\circ$  angular resolution

Claudio Kopper - WIPAC



# detection principle - *track*

$\nu_\mu$  CC-int



factor of  $\approx 2$  energy resolution  
<  $1^\circ$  angular resolution

Claudio Kopper - WIPAC



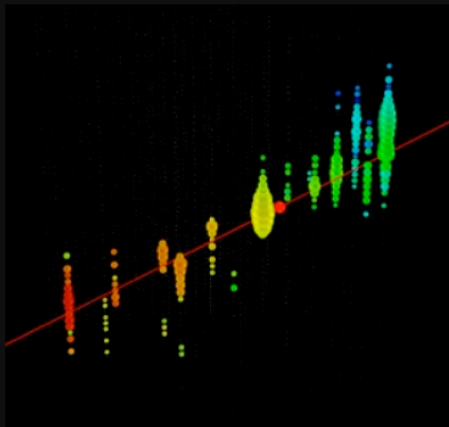
# neutrino detection

## event topologies

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### track

#### CC Muon Neutrino



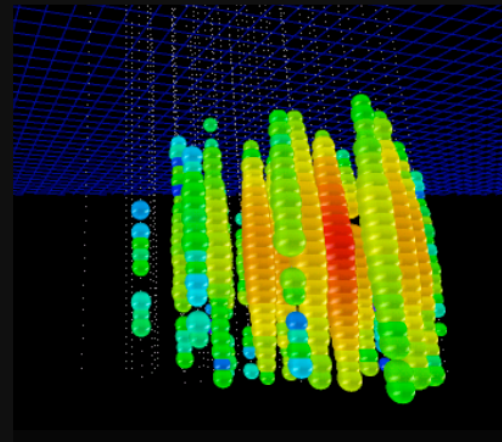
$$\nu_{\mu} + N \rightarrow \mu + X$$

track (data)

factor of  $\approx 2$  energy resolution  
<  $1^{\circ}$  angular resolution

### cascade

#### Neutral Current /Electron Neutrino



$$\nu_e + N \rightarrow e + X$$

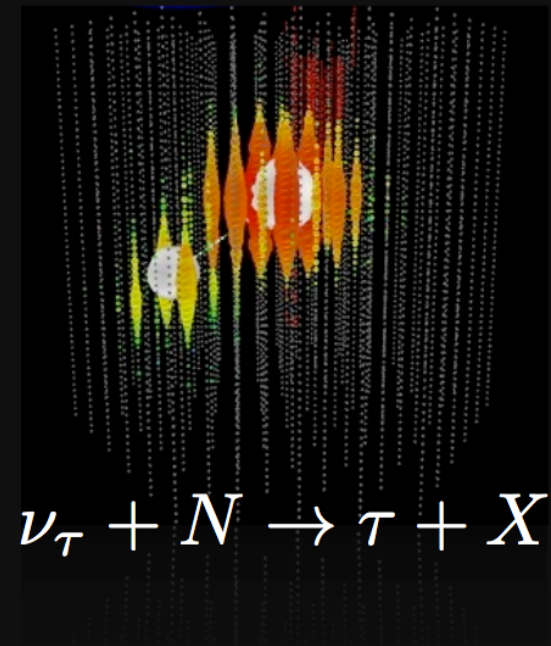
$$\nu_x + N \rightarrow \nu_x + X$$

cascade (data)

$\approx \pm 15\%$  deposited energy resolution  
 $\approx 10^{\circ}$  angular resolution  
(at energies  $\gtrsim 100$  TeV)

### hybrid

#### CC Tau Neutrino



$$\nu_{\tau} + N \rightarrow \tau + X$$

“double-bang” and other signatures  
(simulation)

(not observed yet)

C. Kopper



# THANK YOU

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for any question do not  
hesitate to contact me  
[desiati@wipac.wisc.edu](mailto:desiati@wipac.wisc.edu)

Lectures & Exercises will  
be uploaded to the School portal  
(under Workshops)



# references

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- **printed material**
  - *A Heitler model of extensive air showers* - J. Matthews, *Astrop. Whys.* 22, 287, 2005
  - *Cosmic Rays and Particle Physics* - Thomas K. Gaisser